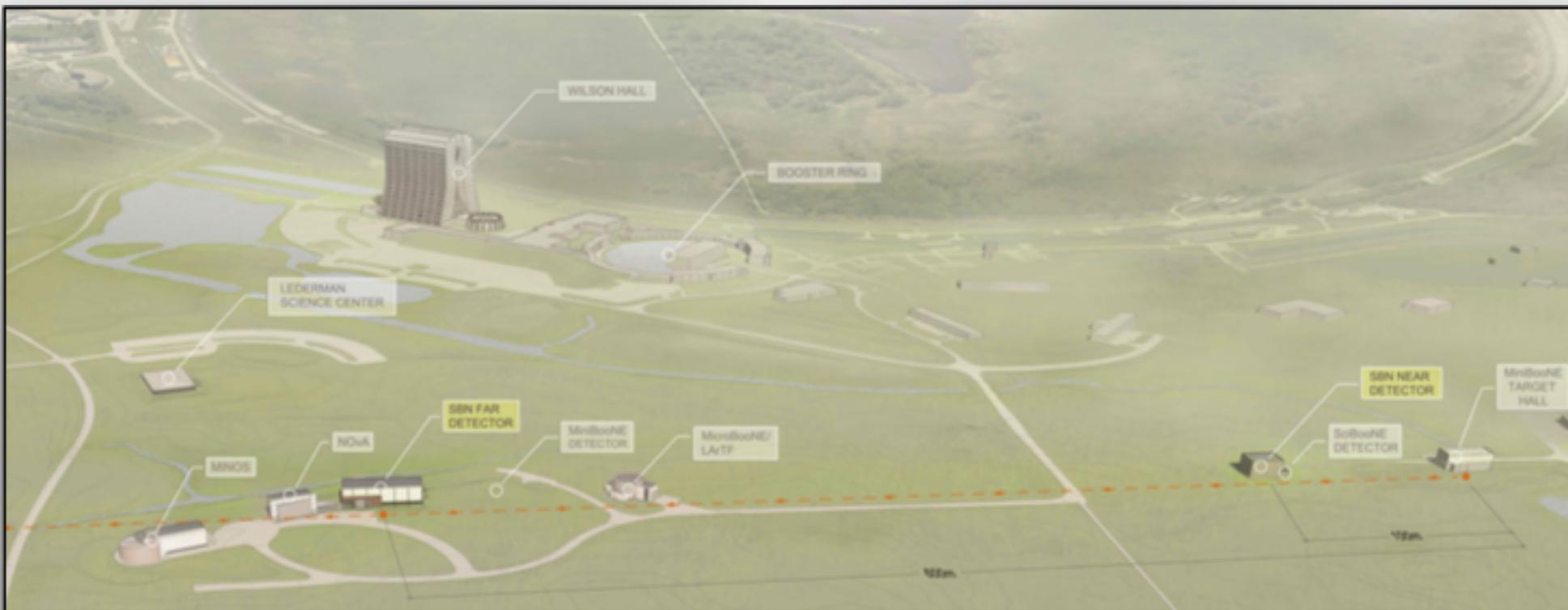


# A short travel for neutrinos in Large Extra Dimensions



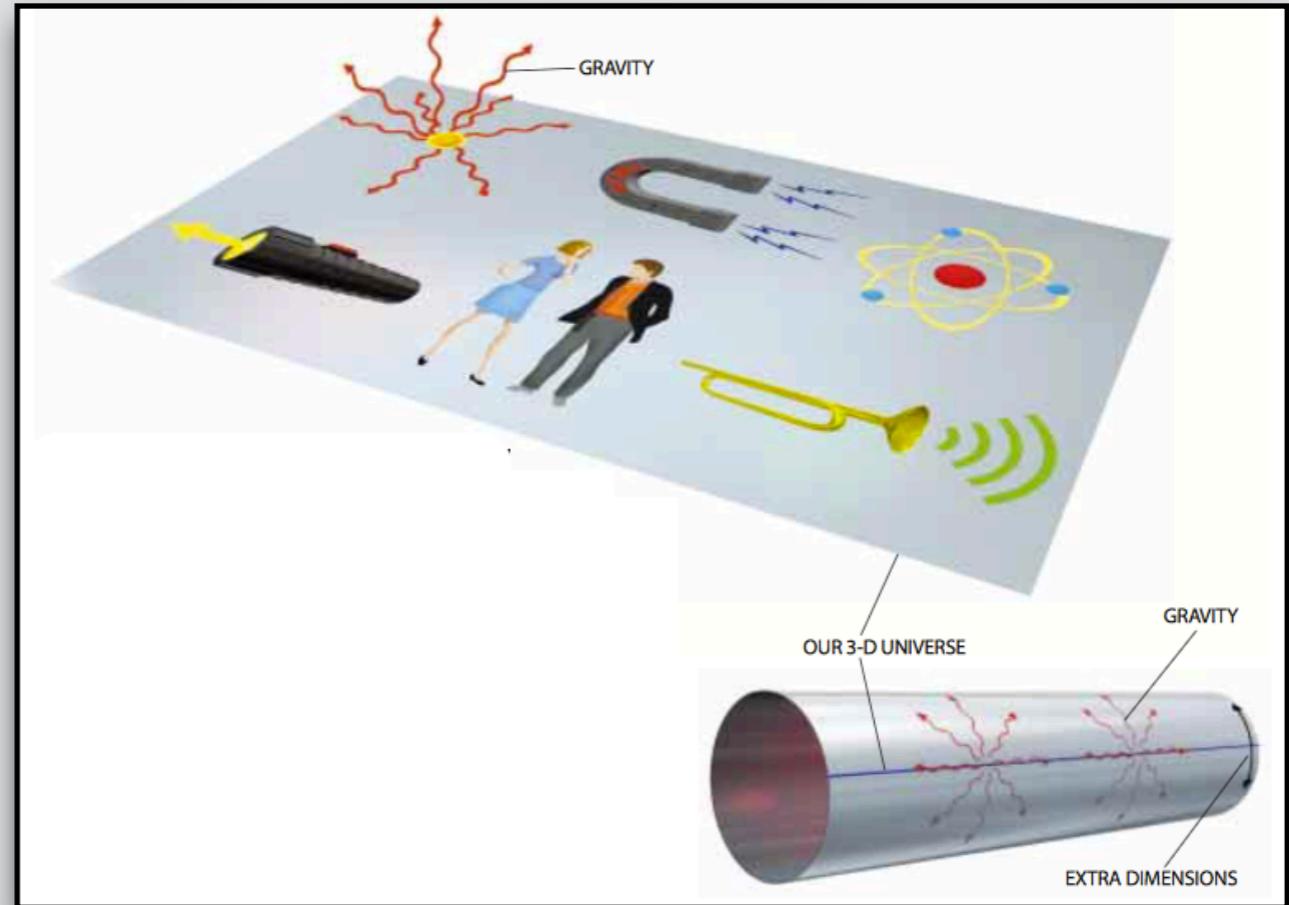
arXiv:1503.01520

Gabriela Vitti Stenico  
Orlando Luis Goulart Peres  
David Vanegas Forero

06/19/2018

# Summary:

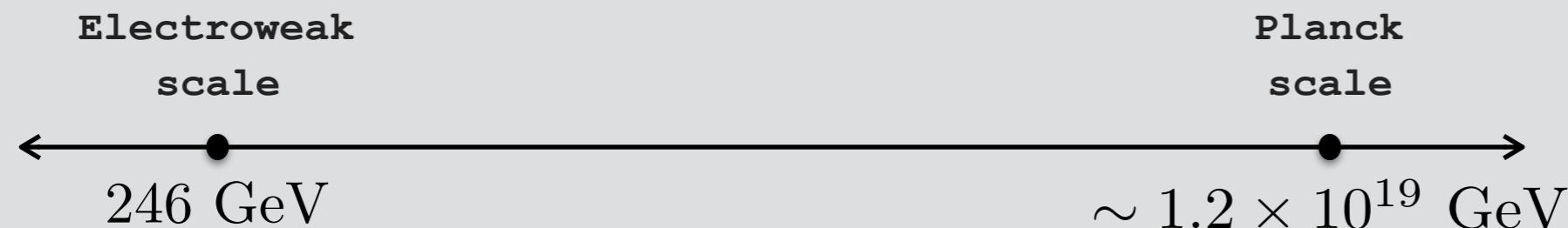
- Why LED?
- Formalism
- Oscillation Probability
- SBN
- Results



Arkani-Hamed, Dimopoulos and Dvali,  
Scientific American, August 2000

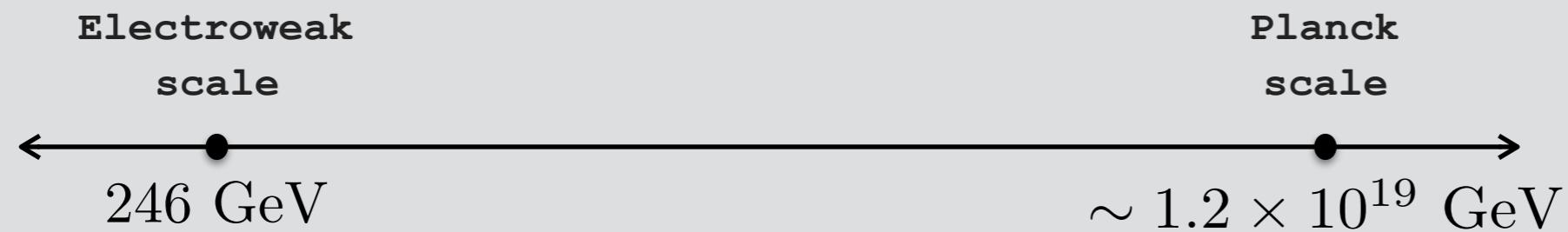
# Why LED?

Hierarchy Problem



# Why LED?

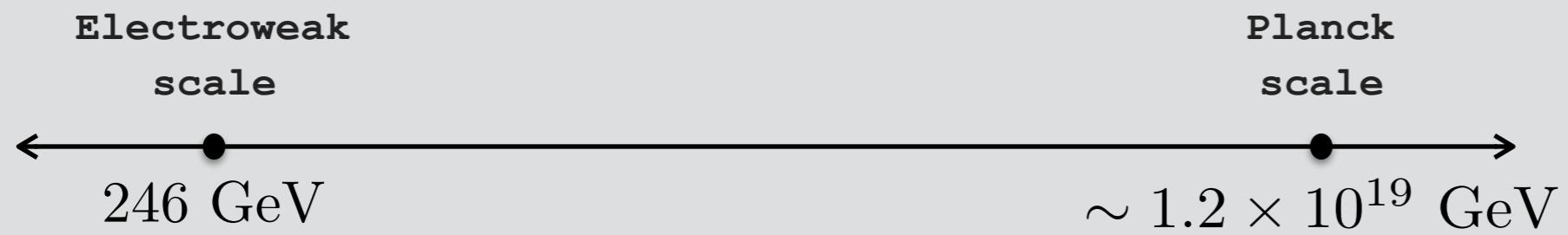
## Hierarchy Problem



- gravity is strong!

# Why LED?

## Hierarchy Problem

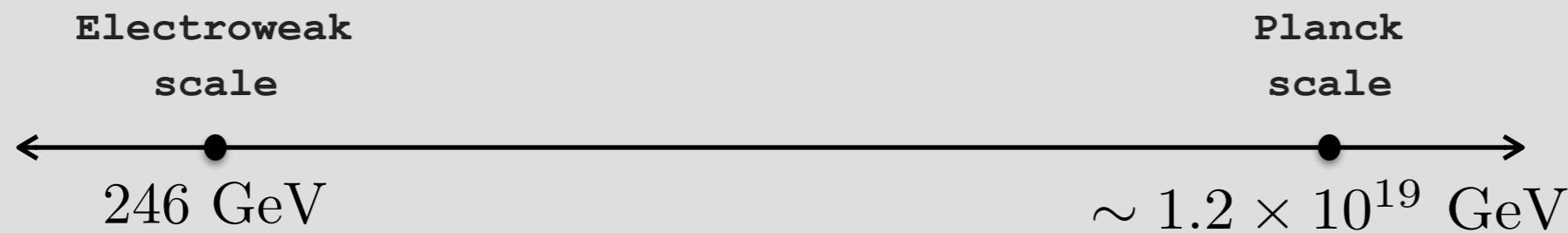


- gravity is strong!

Large disparity

# Why LED?

## Hierarchy Problem



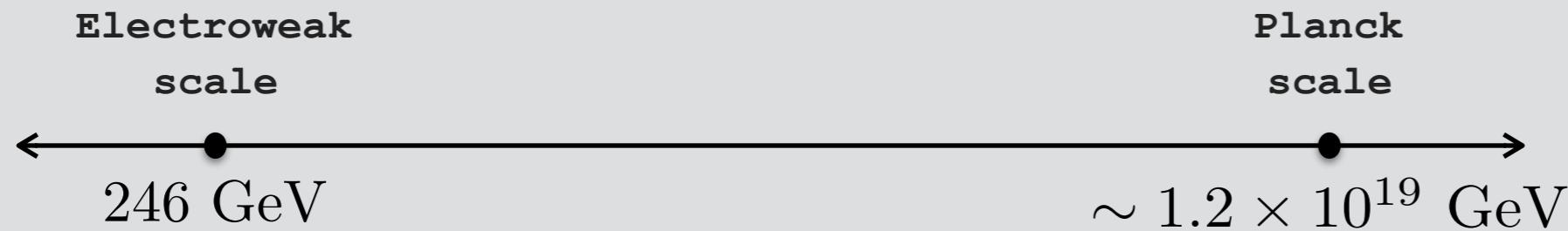
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Large disparity

Solution: graviton can propagate freely in the extra dimensions!

# Why LED?

## Hierarchy Problem



- gravity is strong!

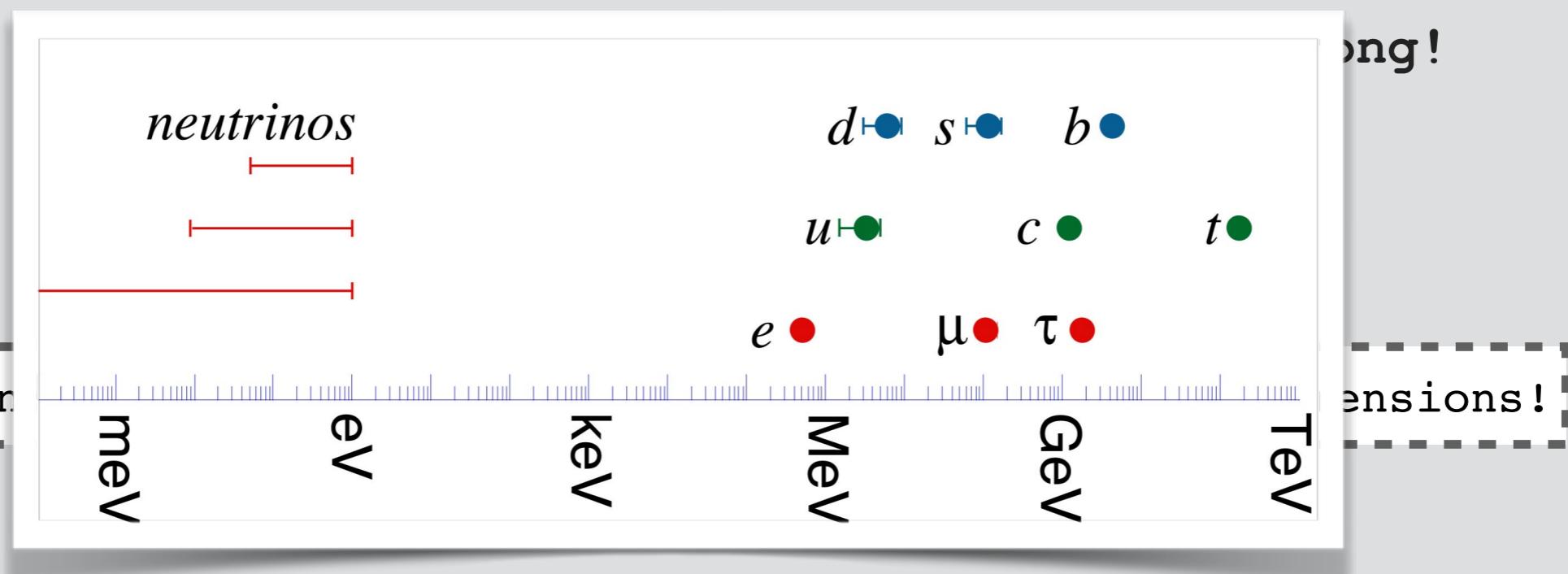
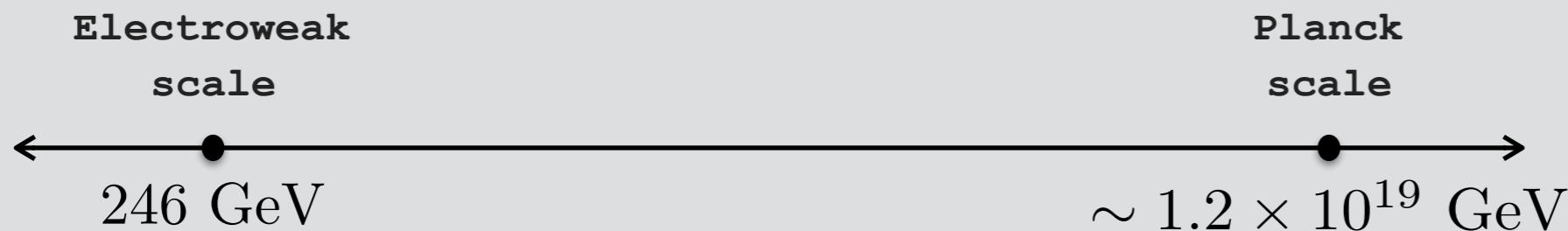
Large disparity

Solution: graviton can propagate freely in the extra dimensions!

*Bonus:* give a natural explanation for the smallness of neutrino masses!

# Why LED?

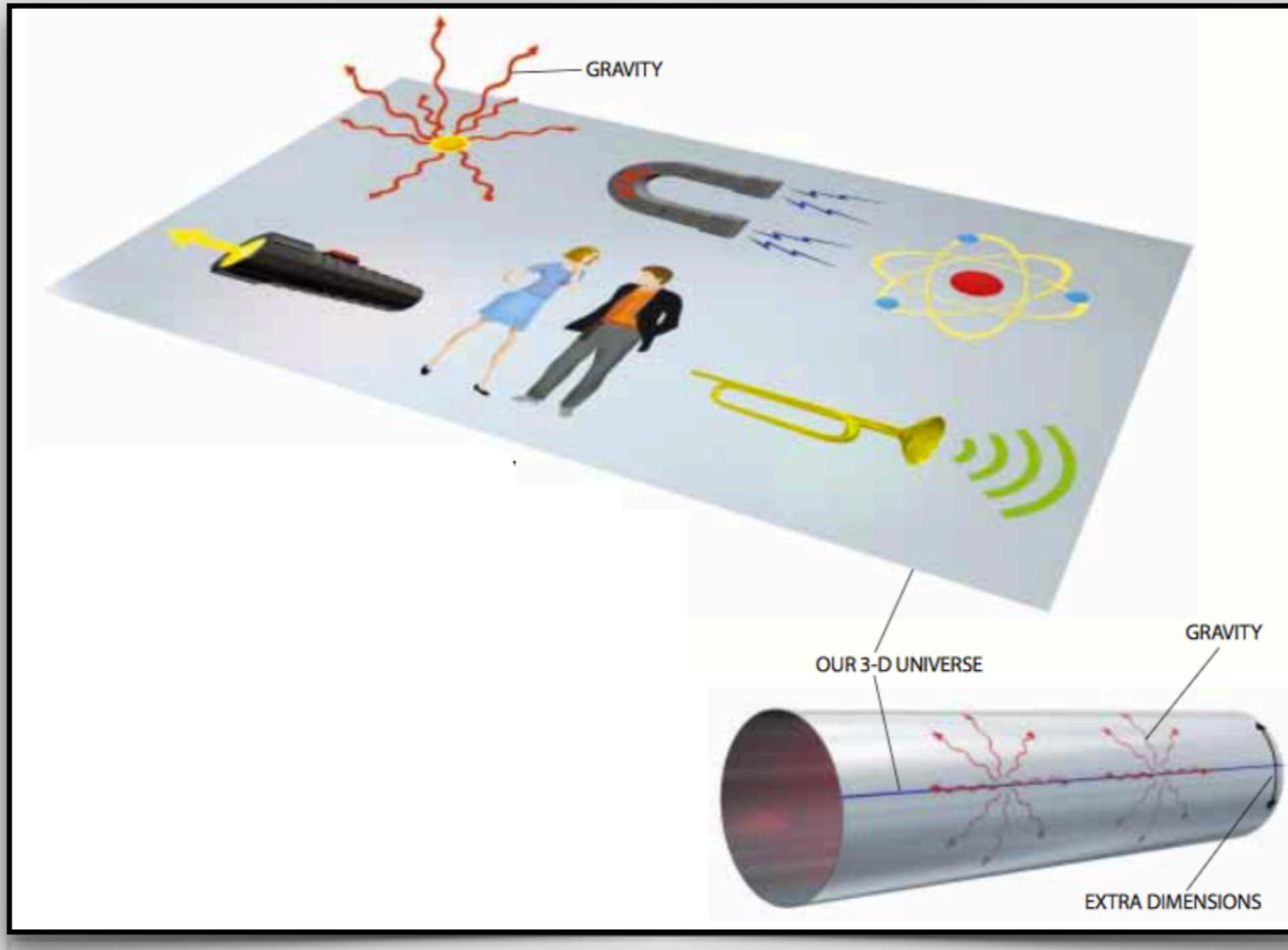
Hierarchy Problem



Bonus: give a natural explanation for the smallness of neutrino masses!

# Formalism

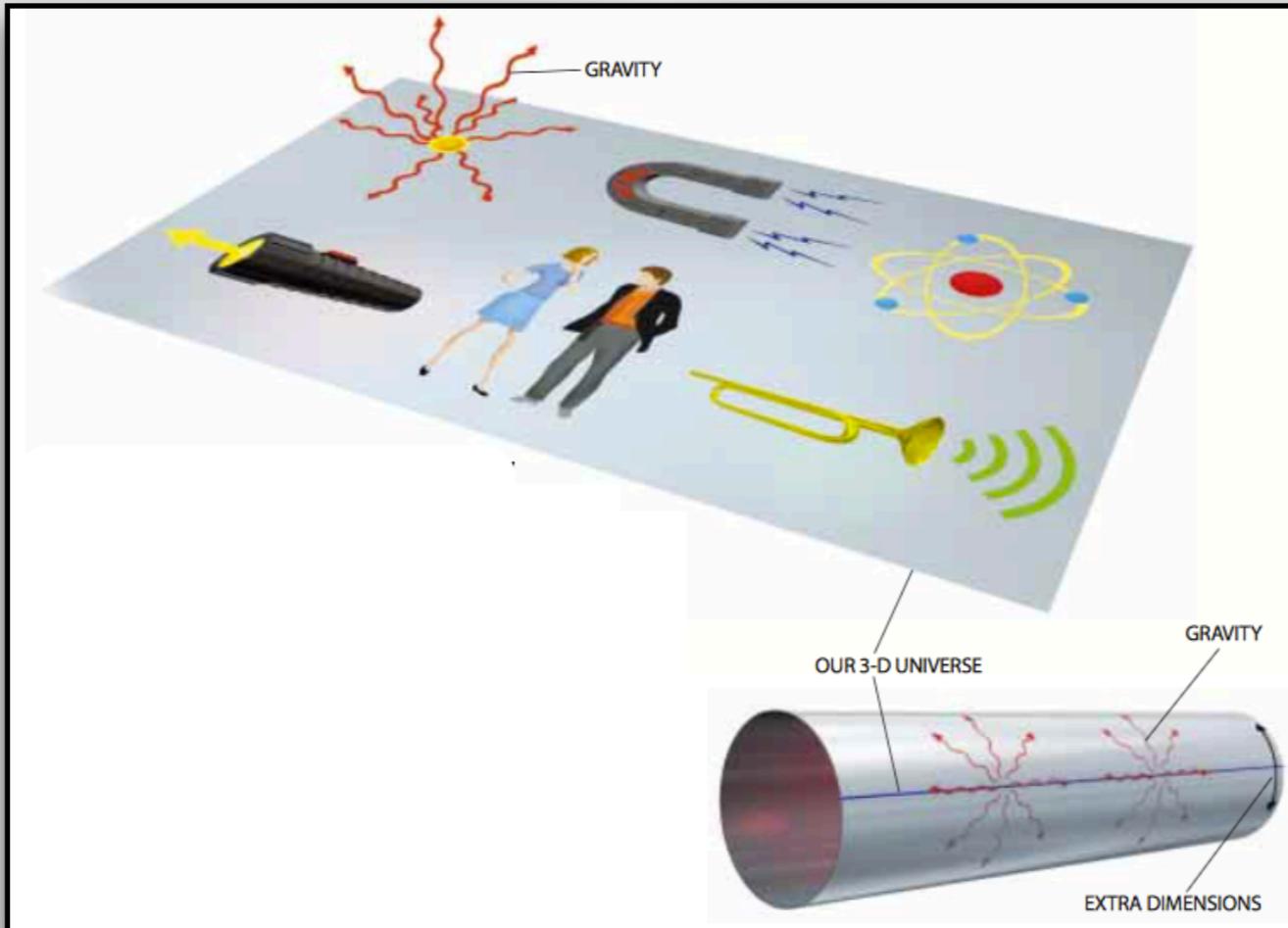
Right-handed neutrino states:  $\Psi_\alpha$  ( $\alpha = e, \mu, \tau$ )



Arkani-Hamed, Dimopoulos and Dvali,  
*Scientific American*, August 2000

# Formalism

Right-handed neutrino states:  $\Psi_\alpha$  ( $\alpha = e, \mu, \tau$ )

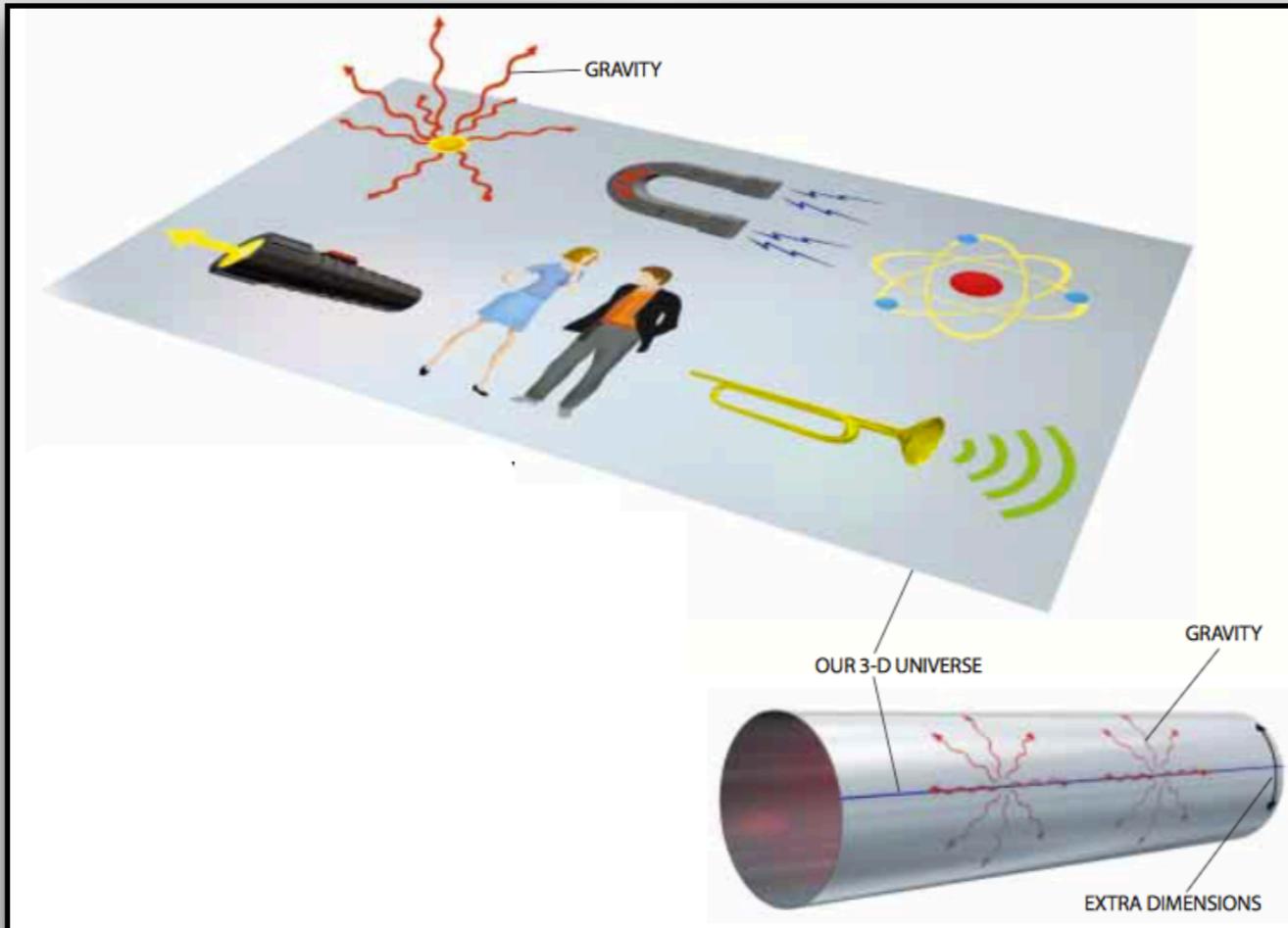


Arkani-Hamed, Dimopoulos and Dvali,  
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D extra Dimensions with compactification radii  $R_j$  ( $j = 1, 2, \dots, D$ )

# Formalism

Right-handed neutrino states:  $\Psi_\alpha$  ( $\alpha = e, \mu, \tau$ )



Arkani-Hamed, Dimopoulos and Dvali,  
Scientific American, August 2000

D extra Dimensions with compactification radii  $R_j$  ( $j = 1, 2, \dots, D$ )

(asymmetric space)

One large spatial scale with radius:  $R_{\text{ED}}$  (5-dimensional space)

## Formalism

*Action of interaction between the active neutrinos and  $\Psi_\alpha$  field is*

$$S_\alpha = \int dx^4 dy i \bar{\Psi}_\alpha \Gamma^A \partial_A \Psi_\alpha + \int dx^4 [i \bar{\nu}_{\alpha L} \gamma^\mu \partial_\mu \nu_{\alpha L} + \kappa_{\alpha\beta} H \bar{\nu}_{\alpha L} \psi_{\beta R}(x, y=0)] + \text{h.c.}$$

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**Extra Dimension**

# Formalism

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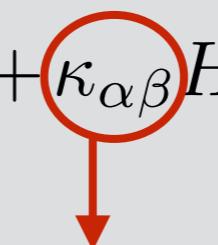


**Higgs doublet**

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**Yukawa coupling matrix**

# Formalism

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**Right-handed neutrino field  
decomposed in  $\Psi_\alpha = (\psi_{\alpha L}, \psi_{\alpha R})$**

## Formalism

*Action of interaction between the active neutrinos and  $\Psi_\alpha$  field is*

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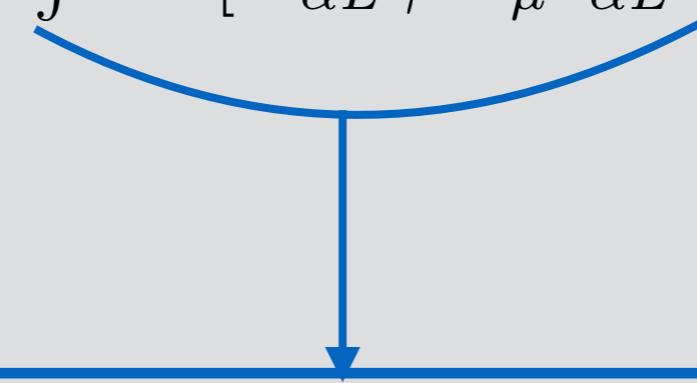
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kinetic term of  $\Psi_\alpha$  field

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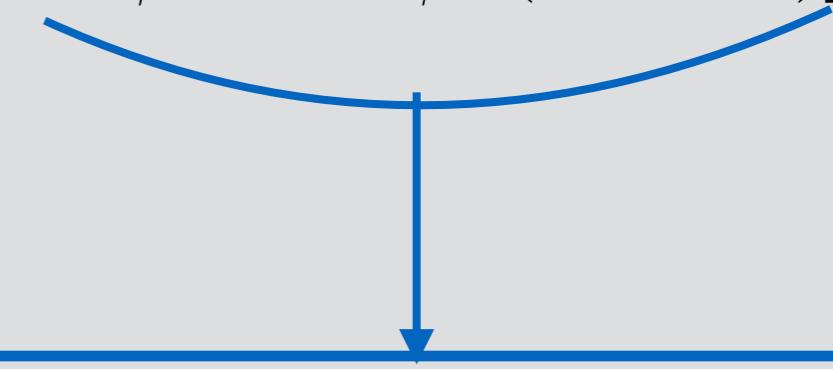


**kinetic term of active neutrinos**

# Formalism

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**Interaction between field  $\Psi_\alpha$  and the active neutrinos**

## Formalism

*Action of interaction between the active neutrinos and  $\Psi_\alpha$  field is*

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After EWSSB:

$$\begin{aligned} \mathcal{L}_{\text{mass}} &= m^{D\alpha\beta} \left( \bar{\nu}_{\alpha R}^{(0)} \nu_{\beta L} + \sqrt{2} \sum_{n=1}^{\infty} \bar{\nu}_{\alpha R}^{(n)} \nu_{\beta L} \right) + \sum_{n=1}^{\infty} \frac{n}{R_{\text{ED}}} \bar{\nu}_{\alpha R}^{(n)} \nu_{\alpha L}^{(n)} + \text{c.h.} \\ &= \sum_{i=1}^3 \bar{\mathcal{N}}_{iR} M_i \mathcal{N}_{iL} + \text{c.h.} \end{aligned}$$

# Formalism

Action of interaction between the active neutrinos and  $\Psi_\alpha$  field is

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## Kaluza-Klein (KK) modes

After EWSSB:

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# Formalism

$$\mathcal{N}_{iL(R)} = \left( \nu_i^{(0)}, \nu_i^{(1)}, \dots \right)_{L(R)}^T \longrightarrow \text{Pseudo mass eigenstates}$$

$$M_i = \begin{bmatrix} m_i^D & 0 & 0 & 0 & \dots \\ \sqrt{2}m_i^D & 1/R_{\text{ED}} & 0 & 0 & \dots \\ \sqrt{2}m_i^D & 0 & 2/R_{\text{ED}} & 0 & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \longrightarrow \text{Need to be diagonalized!}$$

Diagonalizing in the form:  $S_i^\dagger M_i^\dagger M_i S_i$ :

$$\nu_{\alpha L} = \sum_{i=1}^3 U_{\alpha i} \sum_{n=0}^{\infty} S_i^{0n} \nu'_{iL}^{(n)} \quad (\alpha = e, \mu, \tau)$$

# Formalism

The true neutrino masses are:

$$m_i^{(n)} = \frac{\lambda_i^{(n)}}{R_{\text{ED}}}$$

$$\lambda_i^{(n)} - \pi (m_i^D R_{\text{ED}})^2 \cot(\pi \lambda_i^{(n)}) = 0$$

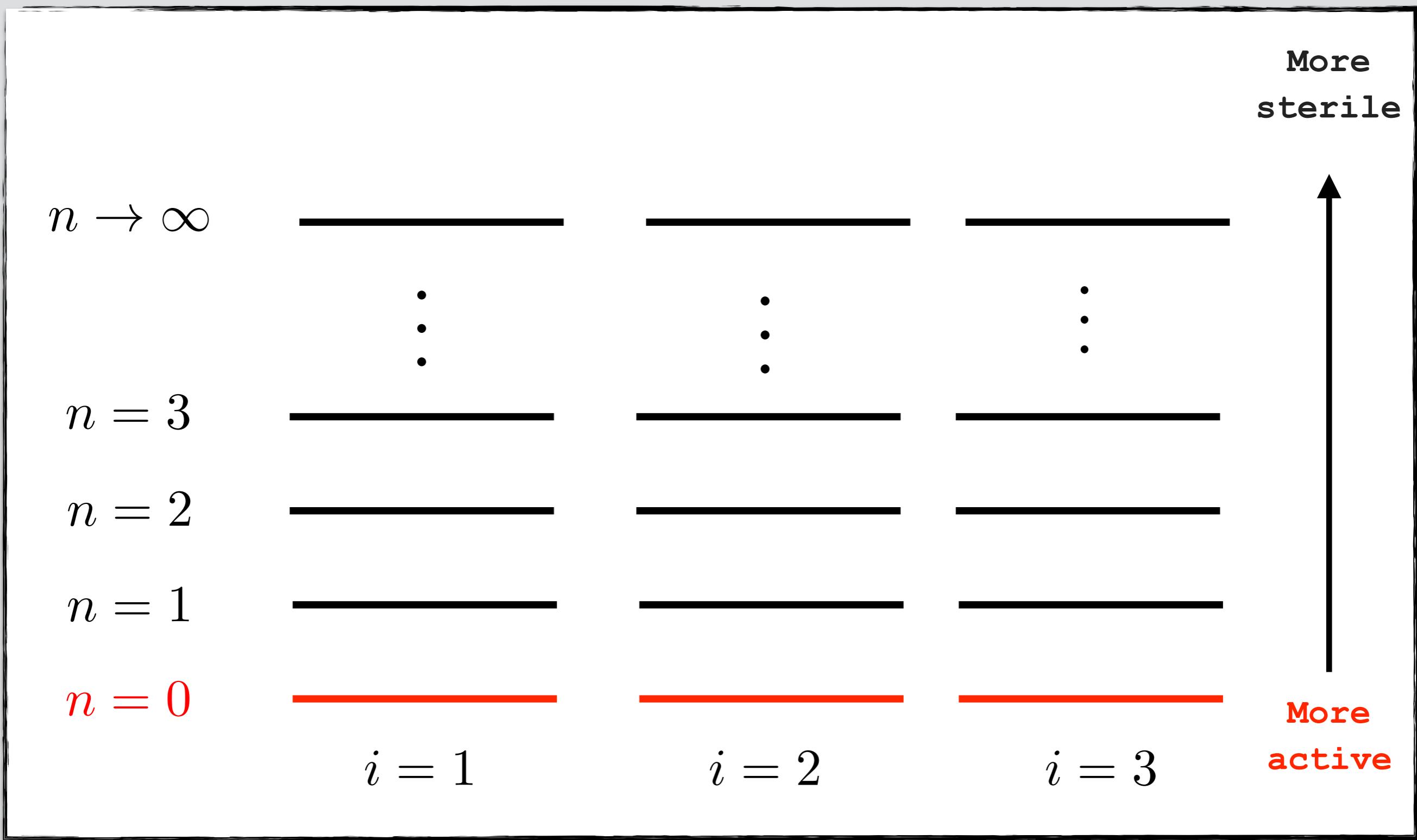
$$n \leq \lambda_i^{(n)} \leq n + 1/2$$

And

$$(S_i^{0n})^2 = \frac{2}{1 + \pi^2 (m_i^D R_{\text{ED}})^2 + (\lambda_i^{(n)})^2 / (m_i^D R_{\text{ED}})^2}.$$

Two new free parameters:  $(m_1^D (m_3^D), R_{\text{ED}})$

# Formalism



## Oscillation Probability

$$P(\nu_\alpha \rightarrow \nu_\beta) = \left| \sum_{i=1}^3 U_{\alpha i} U_{\beta i}^* \sum_{n=0}^{\infty} (S_i^{0n})^2 \exp \left( -i \frac{\lambda_i^{(n)2} L}{2E_\nu R_{\text{ED}}^2} \right) \right|^2$$

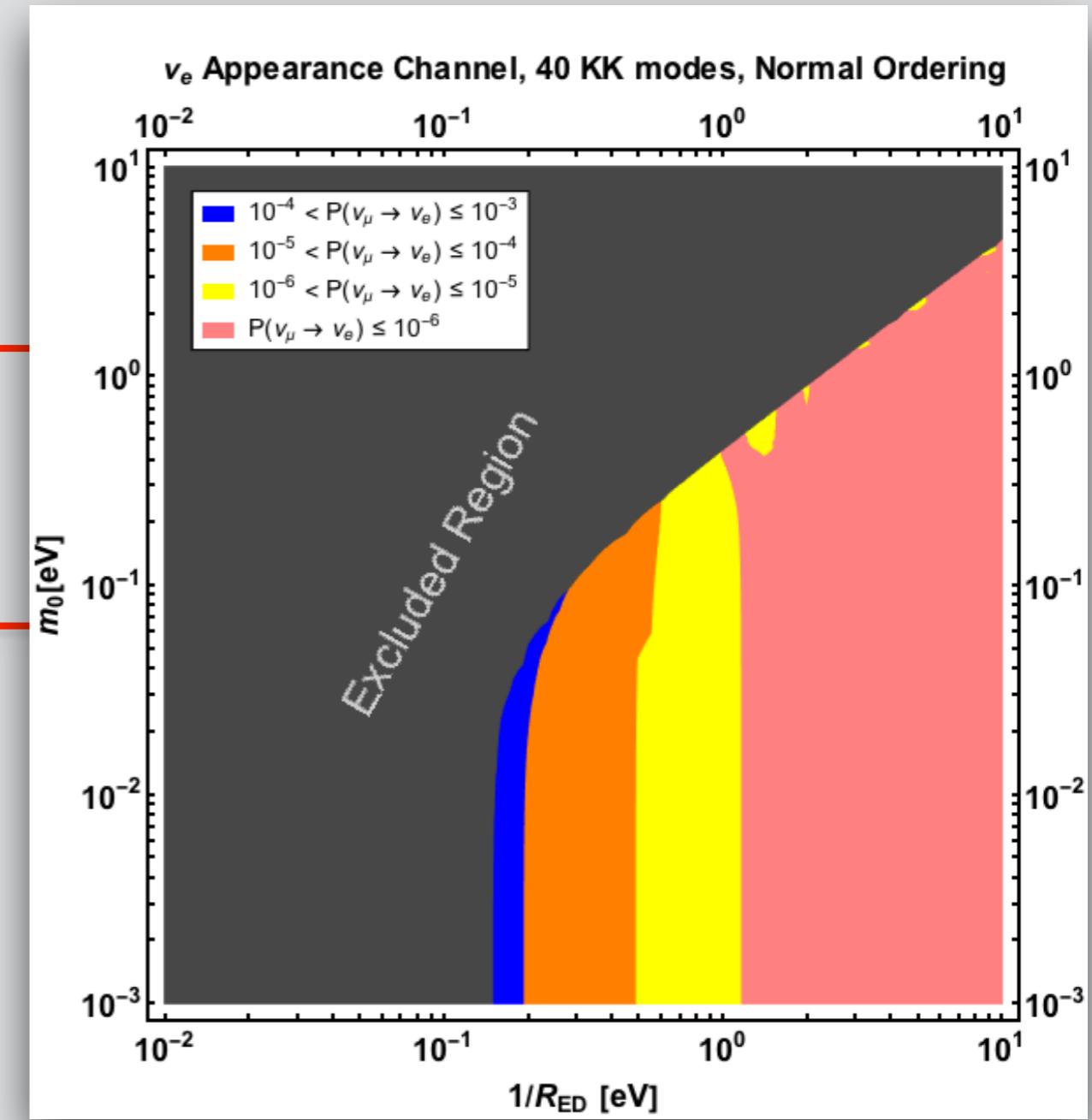
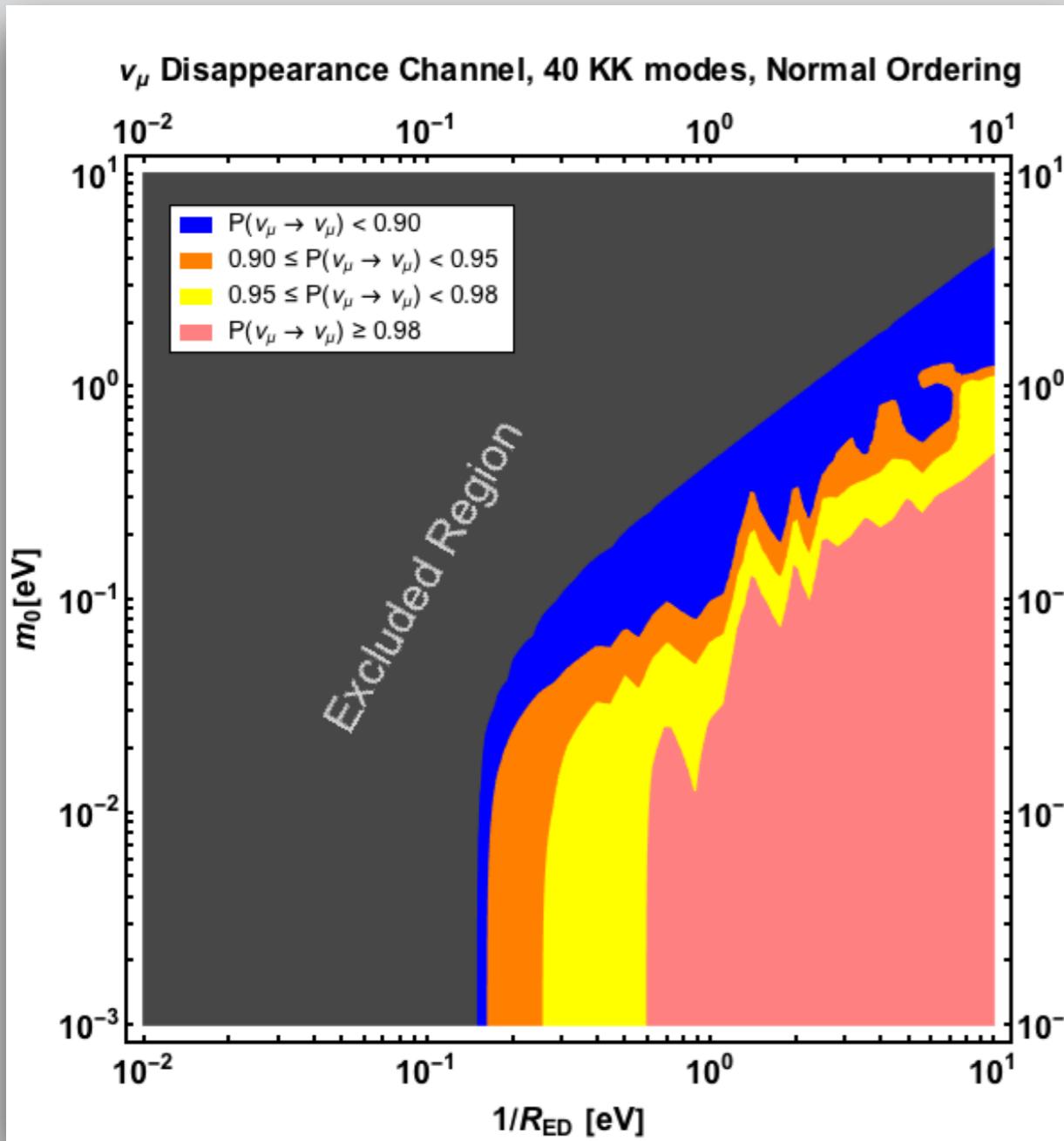
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$$\boxed{\frac{L}{E_\nu} = 1.2 \frac{\text{km}}{\text{GeV}}}$$

# Oscillation Probability

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# Short-Baseline Neutrino Program



arXiv:1503.01520

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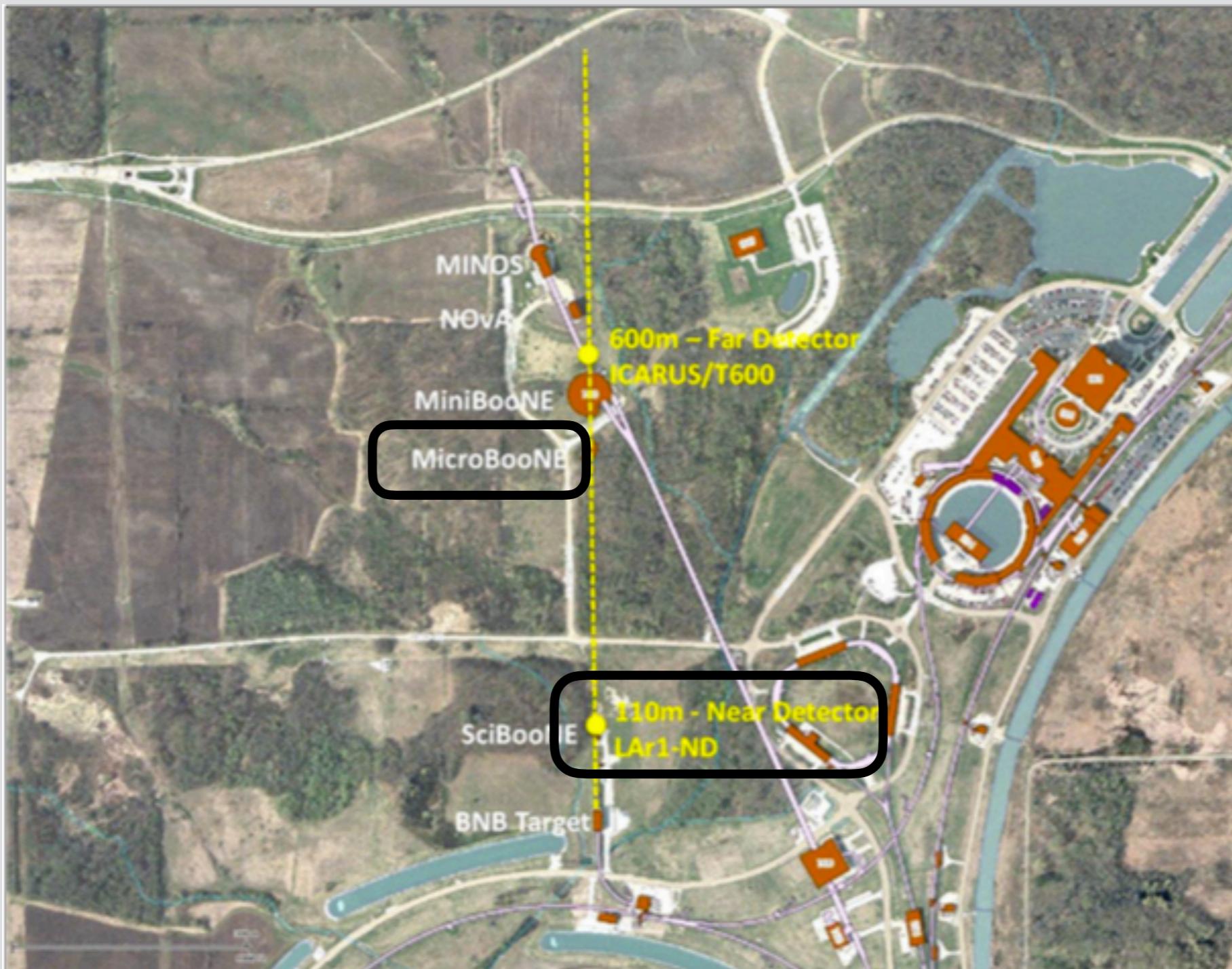
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arXiv:1503.01520

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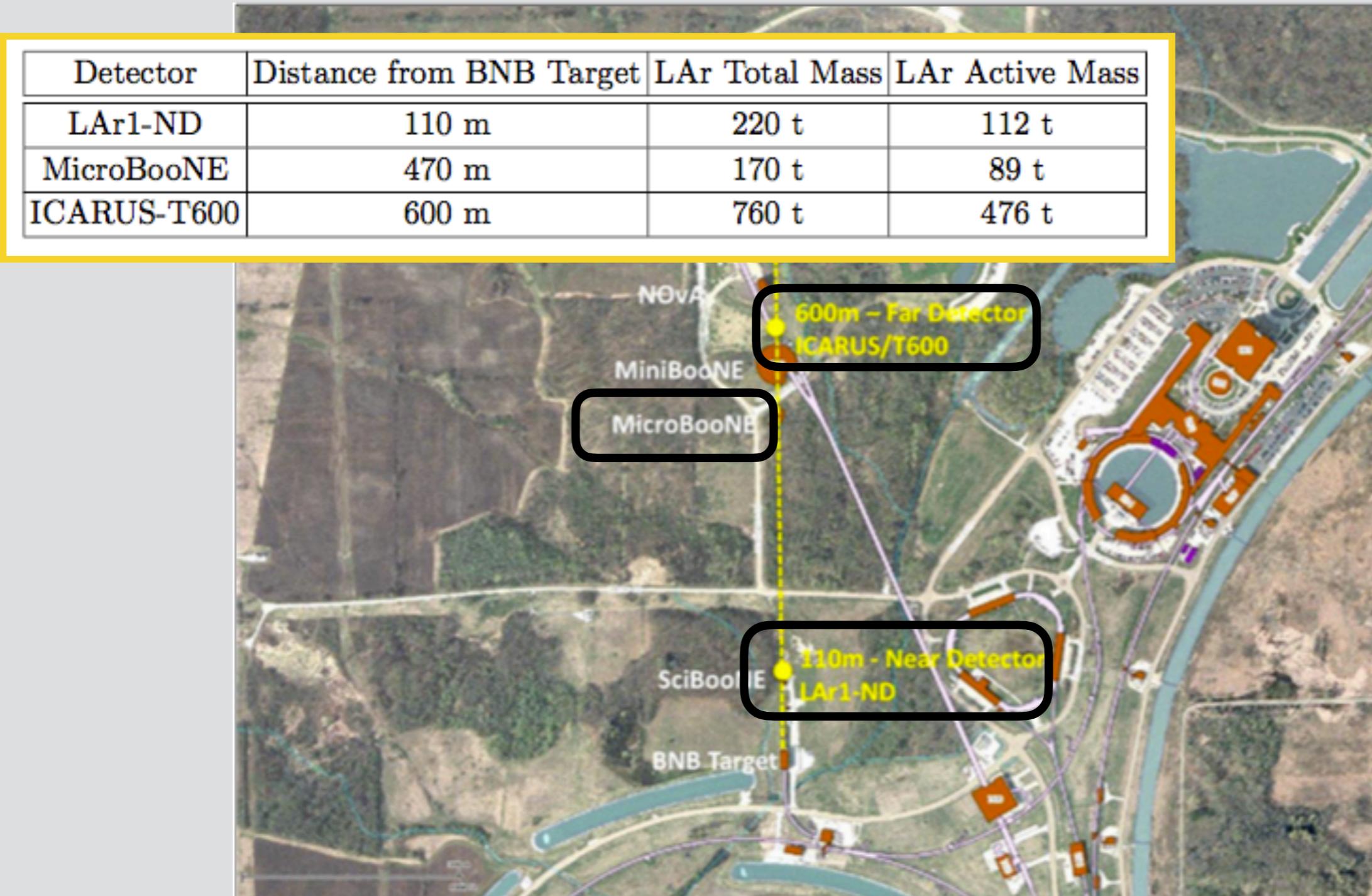
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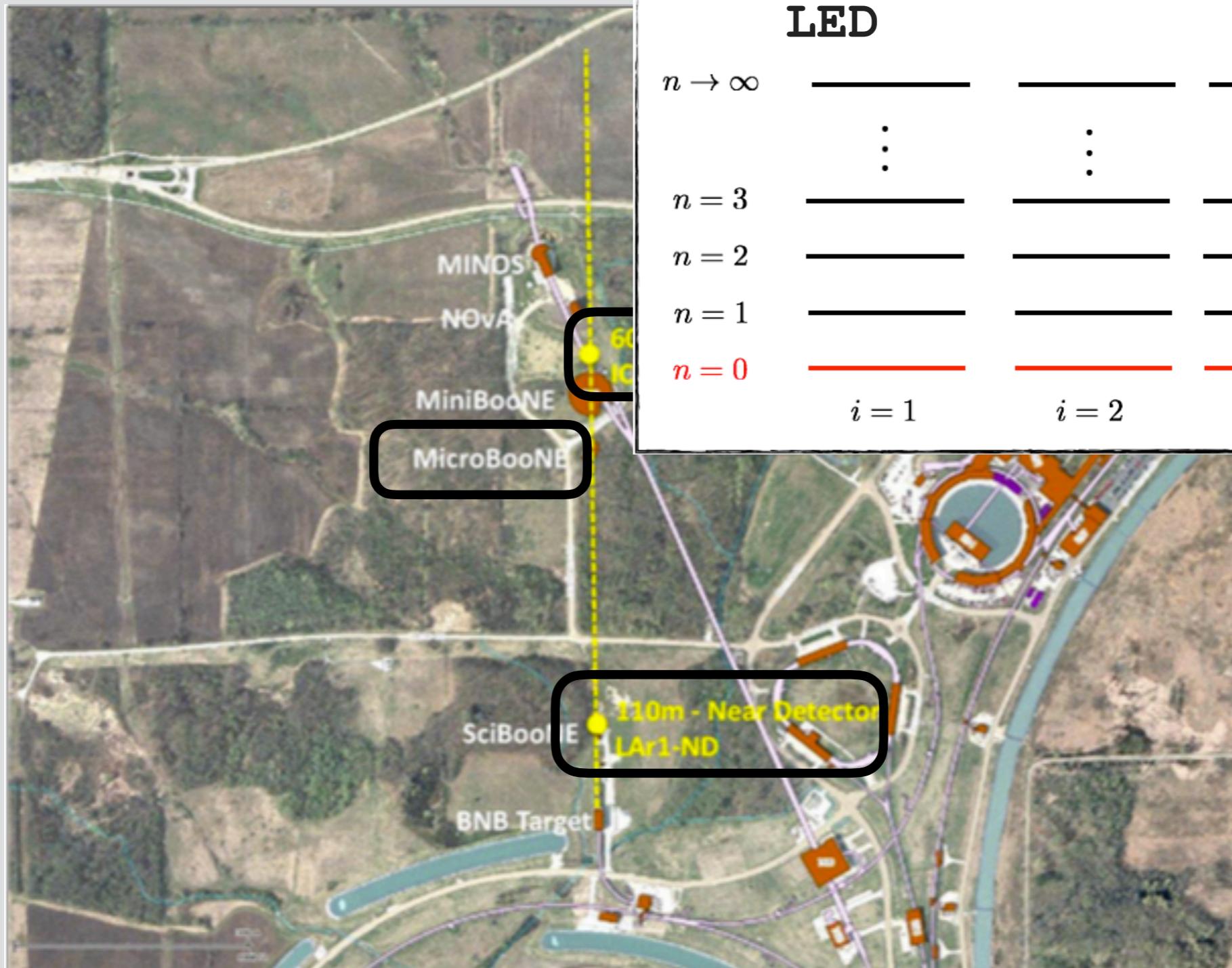
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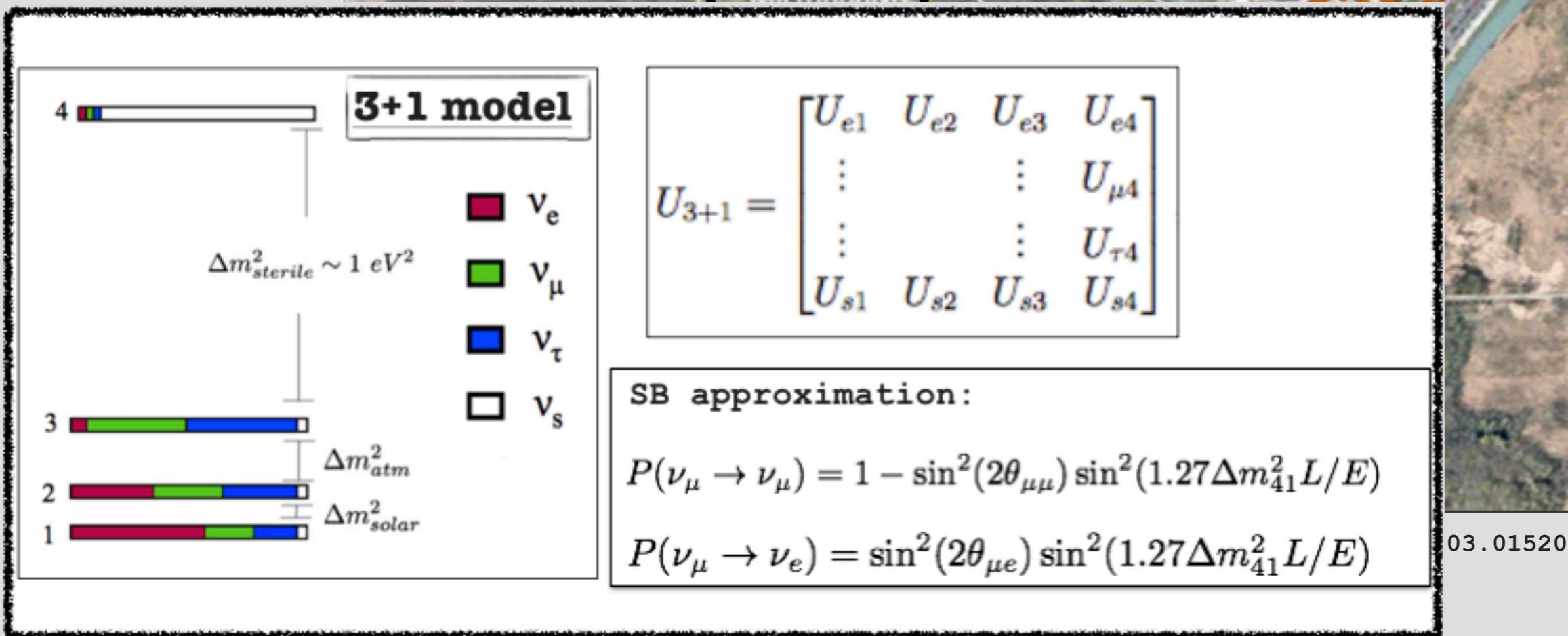
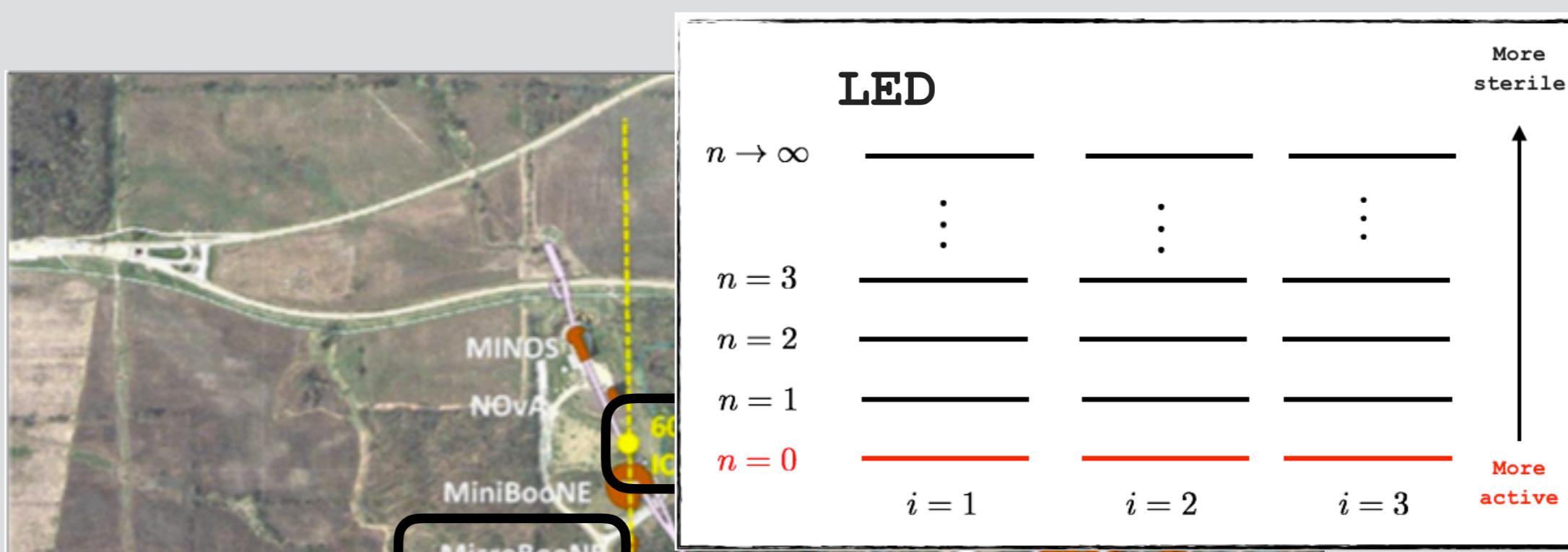
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# Short-Baseline Neutrino Program



arXiv:1503.01520

# Short-Baseline Neutrino Program



## Results:

**GLoBES**

+

arXiv:1503.01520v1

A Proposal for a Three Detector  
Short-Baseline Neutrino Oscillation Program  
in the Fermilab Booster Neutrino Beam

arXiv:1503.01520

NH

IH

# Results:

GLoBES

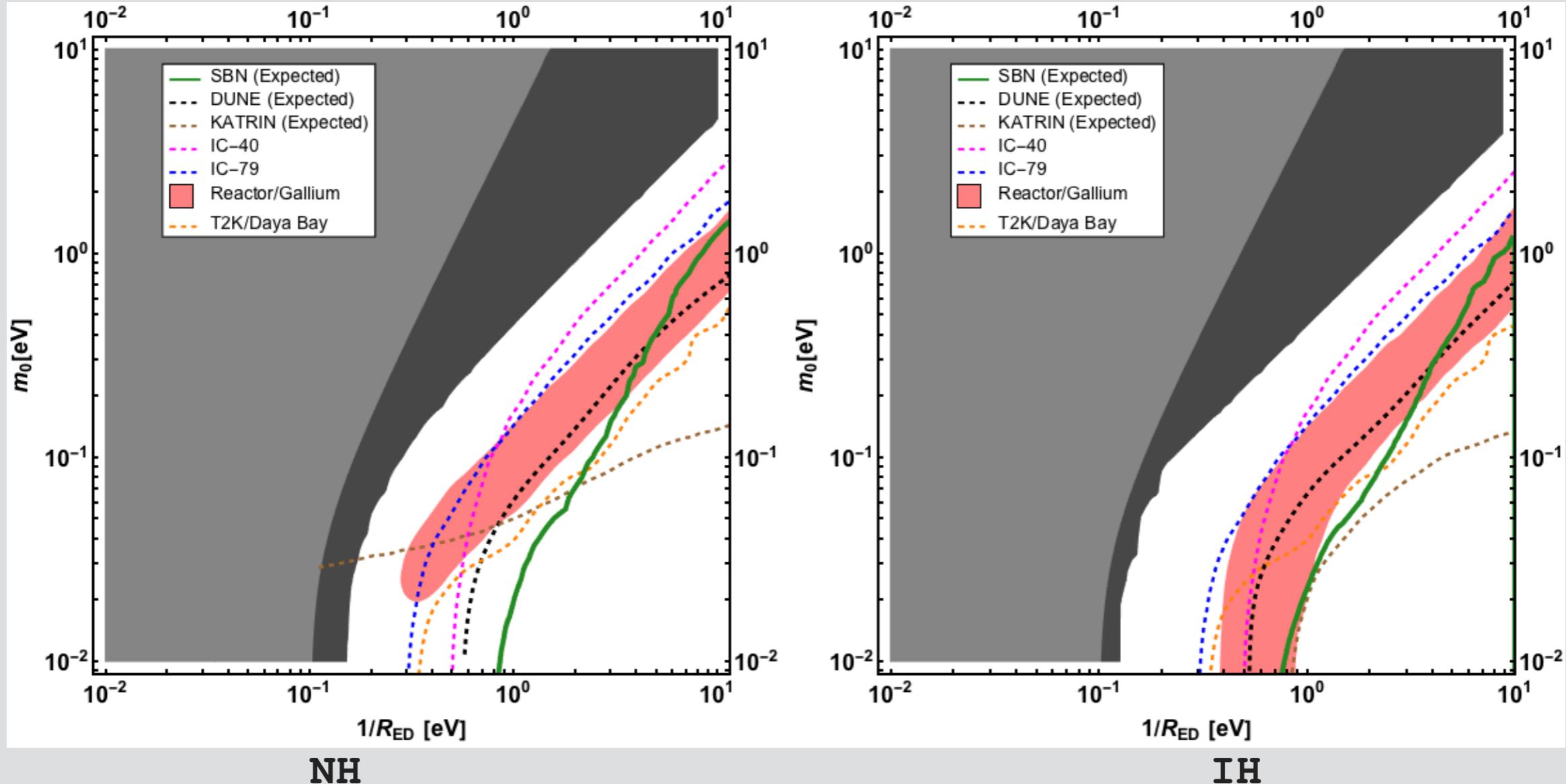
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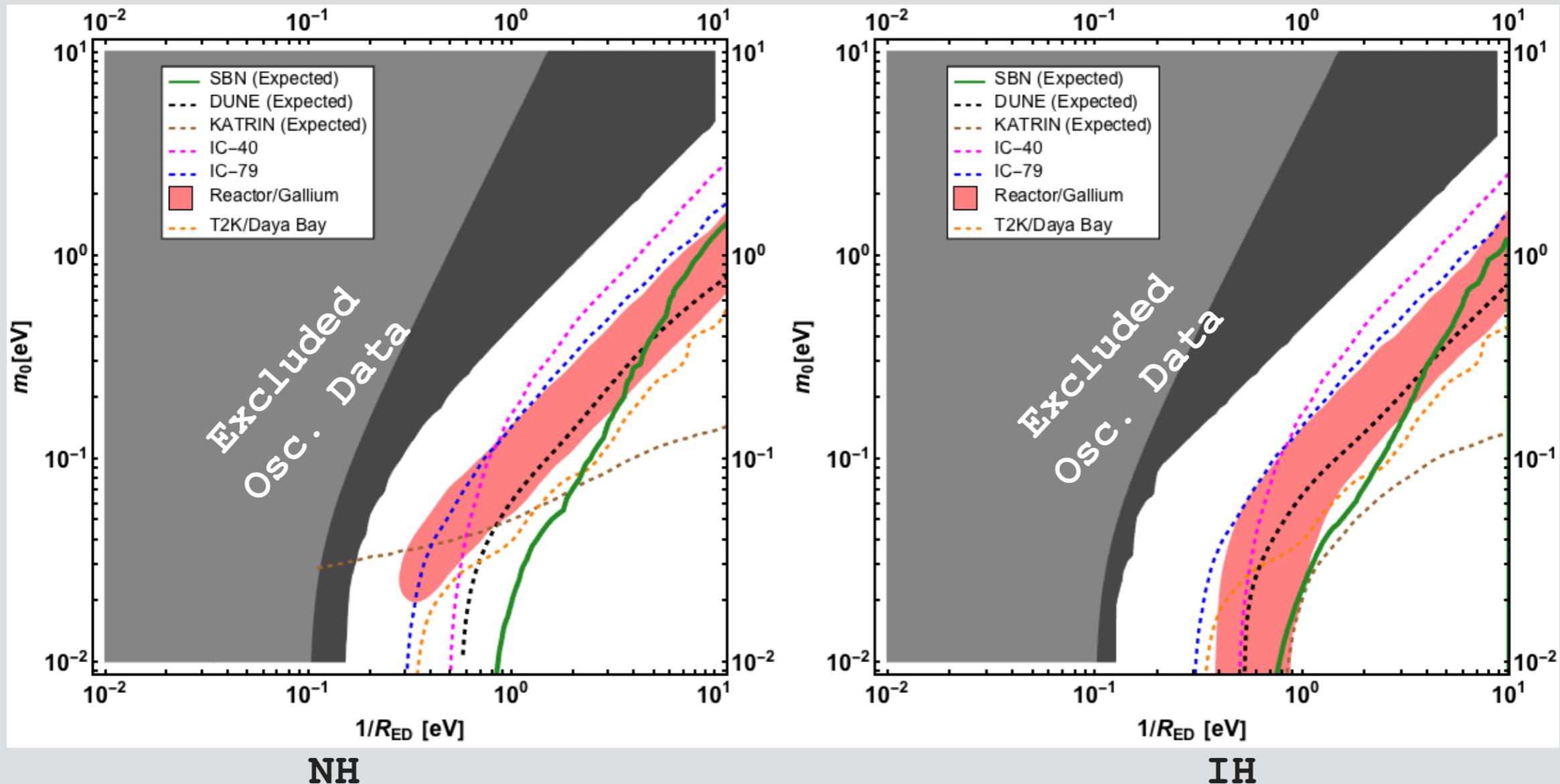
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A Proposal for a Three Detector  
Short-Baseline Neutrino Oscillation Program  
in the Fermilab Booster Neutrino Beam

Sensitivity analysis 90% CL

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## Constrains by neutrino oscillation data:

Square mass differences:

$$\frac{\left(\lambda_2^{(0)}\right)^2 - \left(\lambda_1^{(0)}\right)^2}{R_{\text{ED}}^2} = \Delta m_{\text{sol}}^2 \quad \text{and} \quad \left| \frac{\left(\lambda_3^{(0)}\right)^2 - \left(\lambda_1^{(0)}\right)^2}{R_{\text{ED}}^2} \right| = \Delta m_{\text{atm}}^2.$$

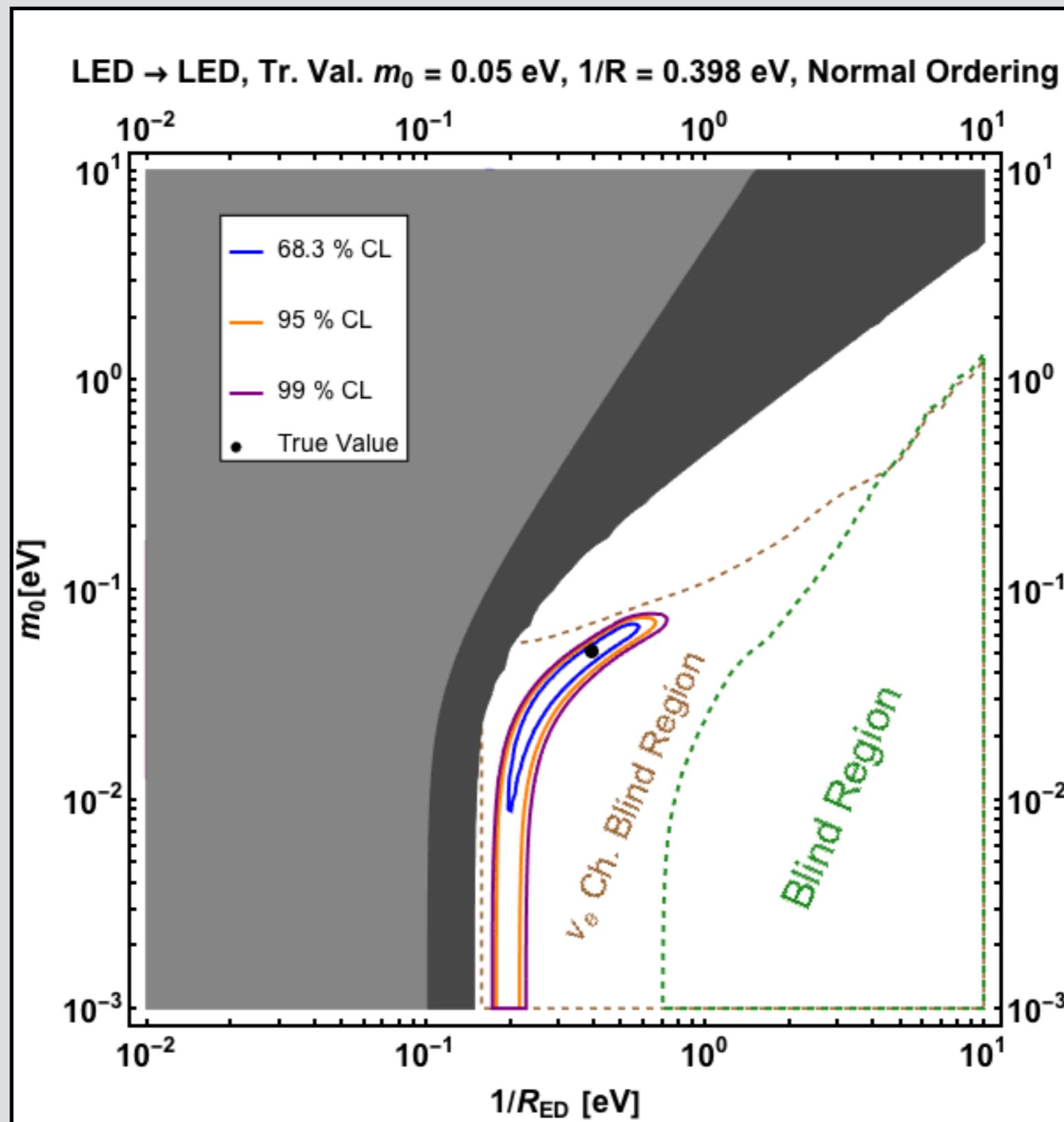
Mixing angles:

$$\sin \theta_{13}^{\text{LED}} = \frac{\sin \theta_{13}^{3\nu}}{(S_3^{00})}$$

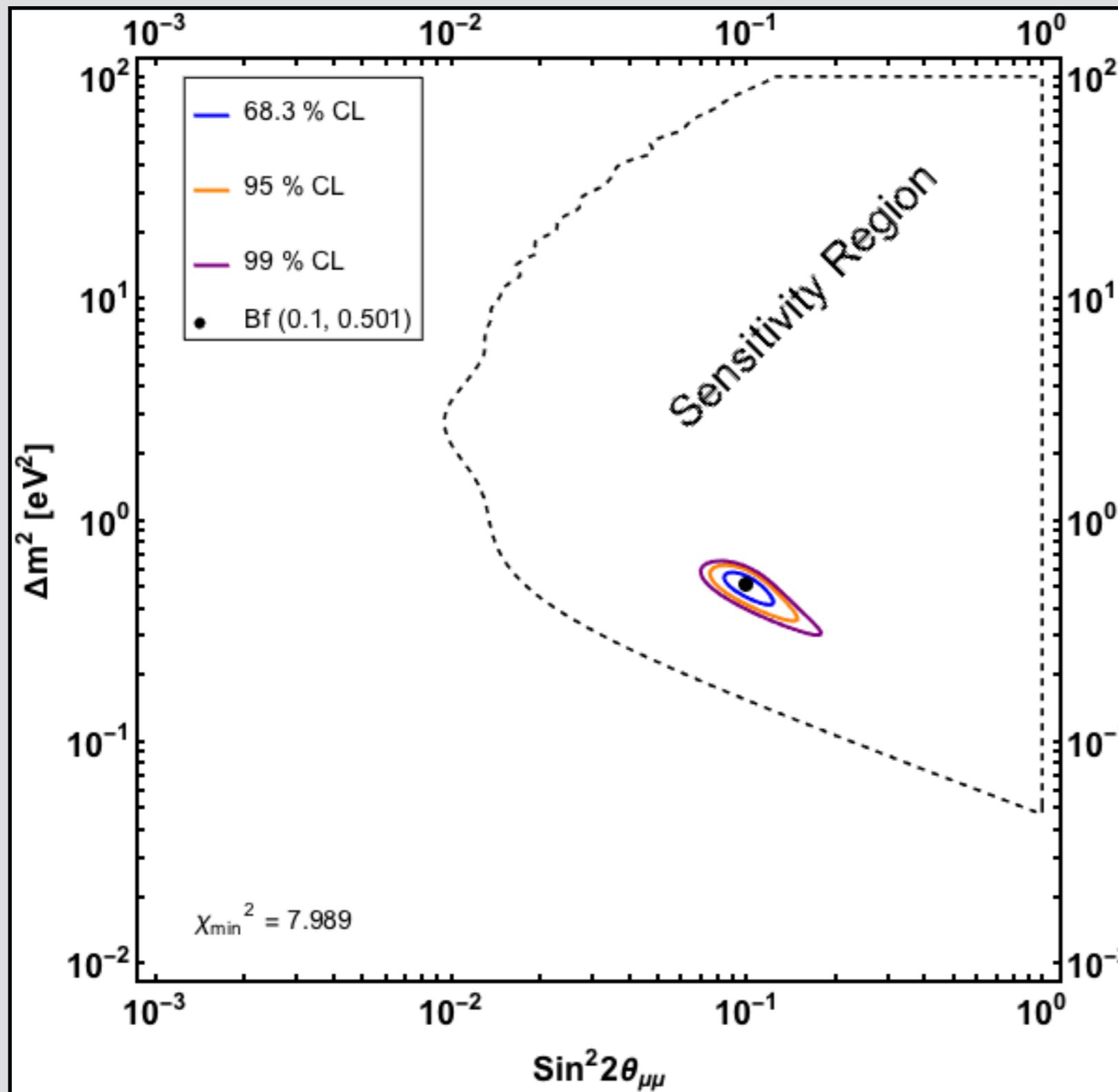
$$\cos \theta_{13}^{\text{LED}} \sin \theta_{12}^{\text{LED}} = \frac{\cos \theta_{13}^{3\nu} \sin \theta_{12}^{3\nu}}{(S_2^{00})}$$

$$\cos \theta_{13}^{\text{LED}} \sin \theta_{23}^{\text{LED}} = \frac{\cos \theta_{13}^{3\nu} \sin \theta_{23}^{3\nu}}{(S_3^{00})}$$

## SBN potential to measure LED parameters



## 3+1 fit to the LED scenario





# Thank you!

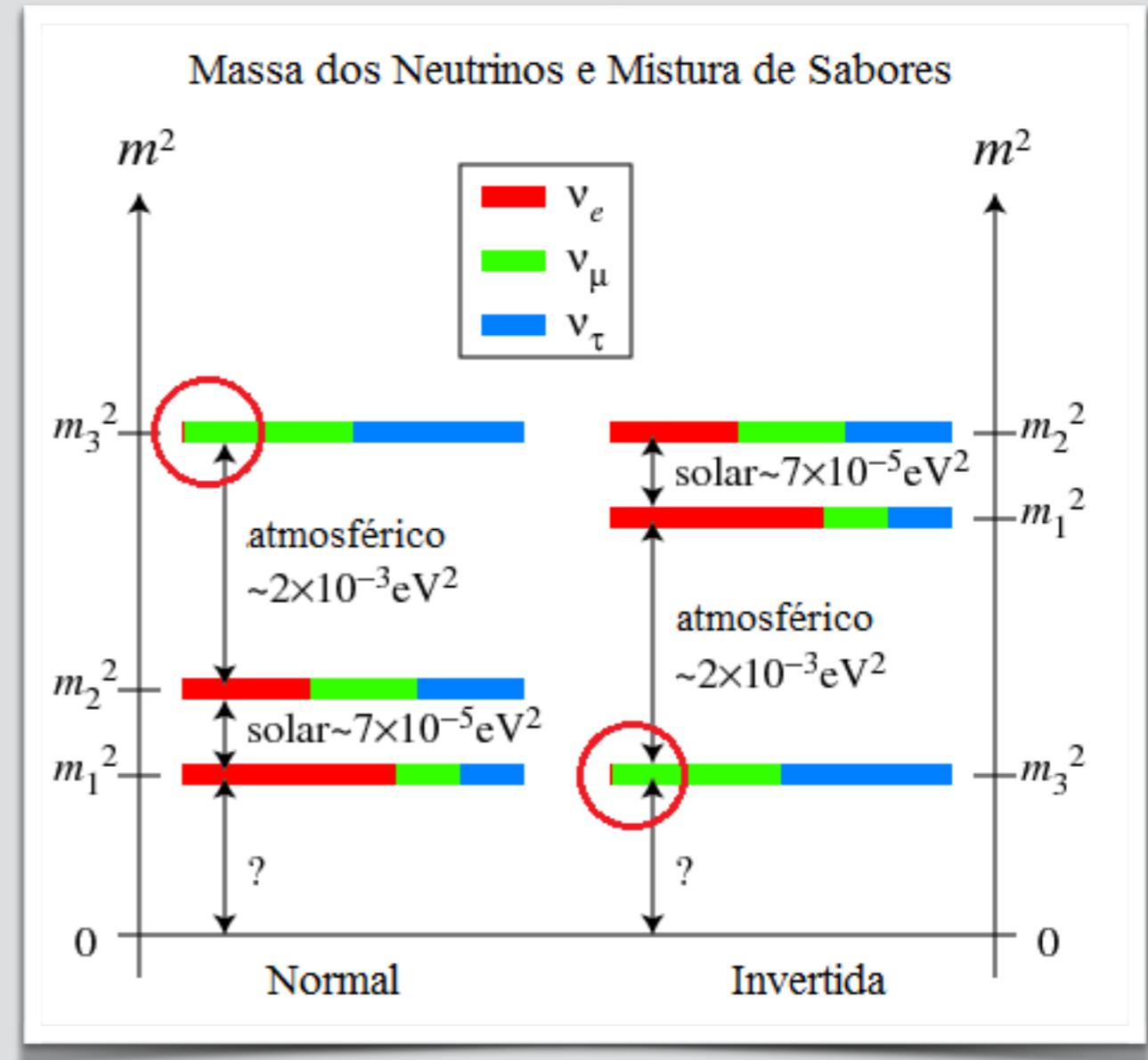


# Neutrino Oscillation

$$|\nu_i\rangle = \sum_{\alpha} U_{\alpha} |\nu_{\alpha}\rangle$$

$$|\nu_{\alpha}\rangle = \sum_i U_{i\alpha}^* |\nu_i\rangle$$

$i$  ( $\alpha = e, \mu, \tau$  and  $i = 1, 2, 3$ )



$$x_{\text{osc}} = \frac{4\pi E}{\Delta m_{ij}^2}$$

$$P(\nu_{\alpha} \rightarrow \nu_{\beta}) = \sum_{i,j} U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^* \exp \left( i \frac{m_j^2 - m_i^2}{2E} x \right)$$

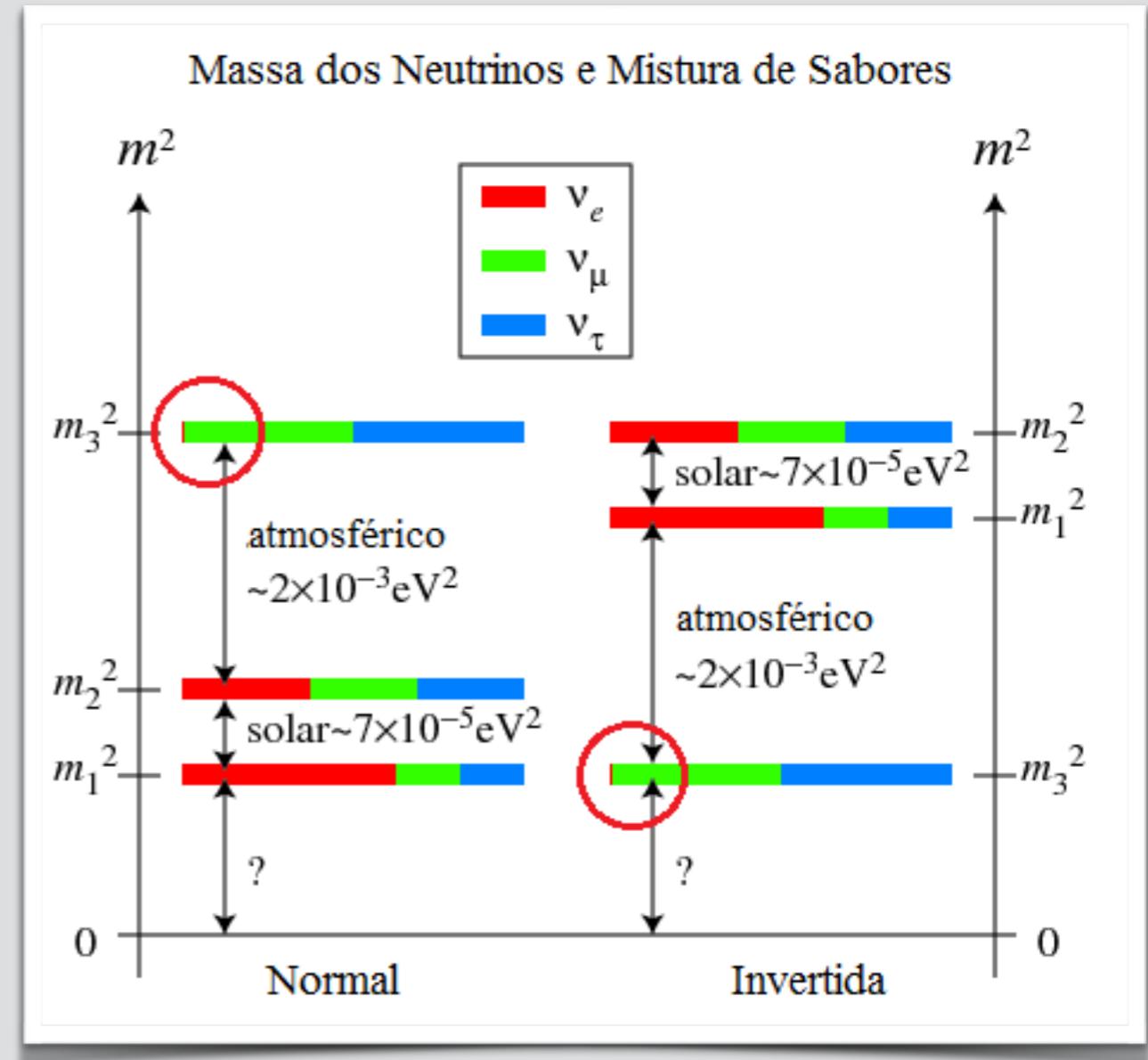
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$\nu_e$

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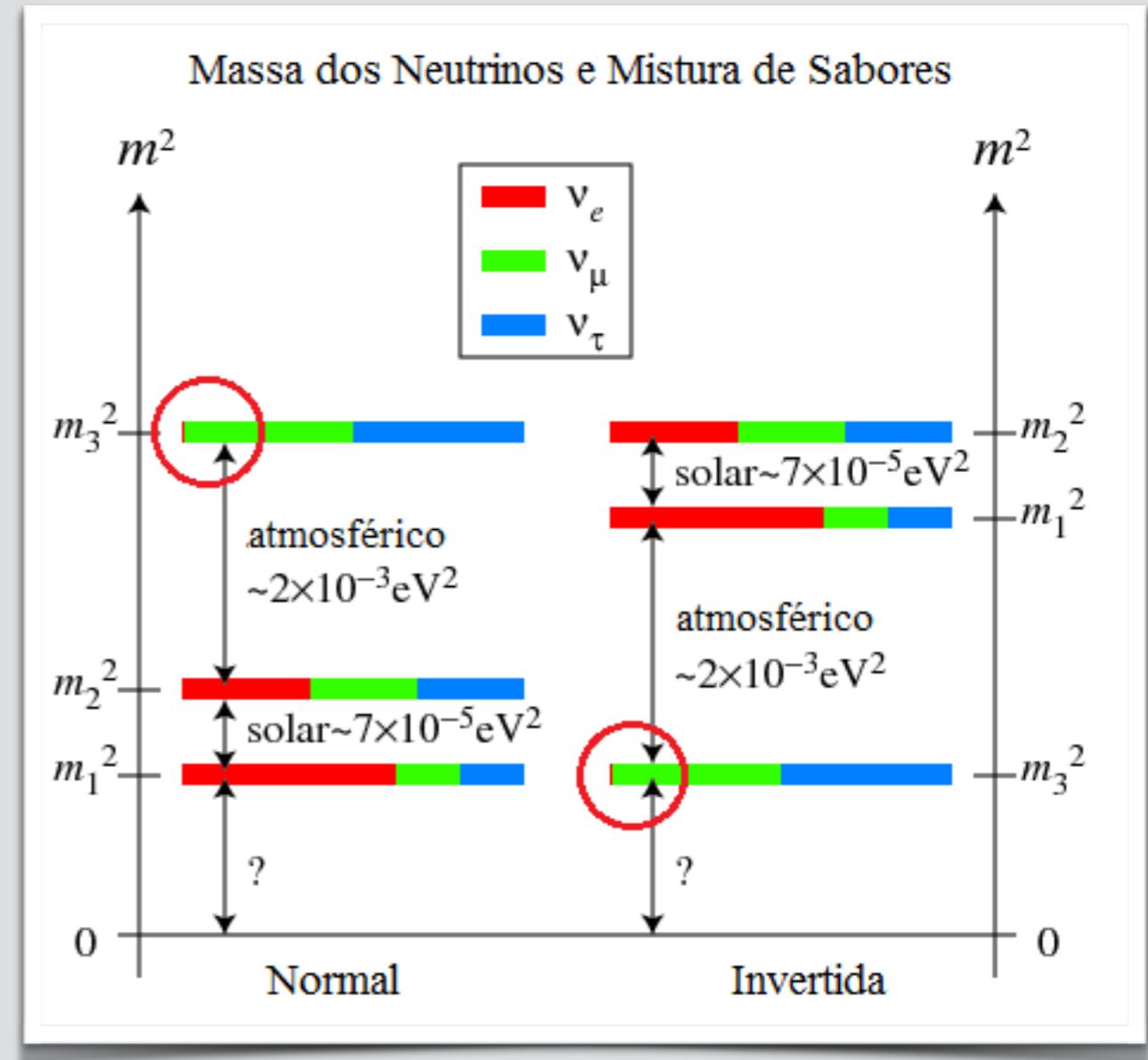
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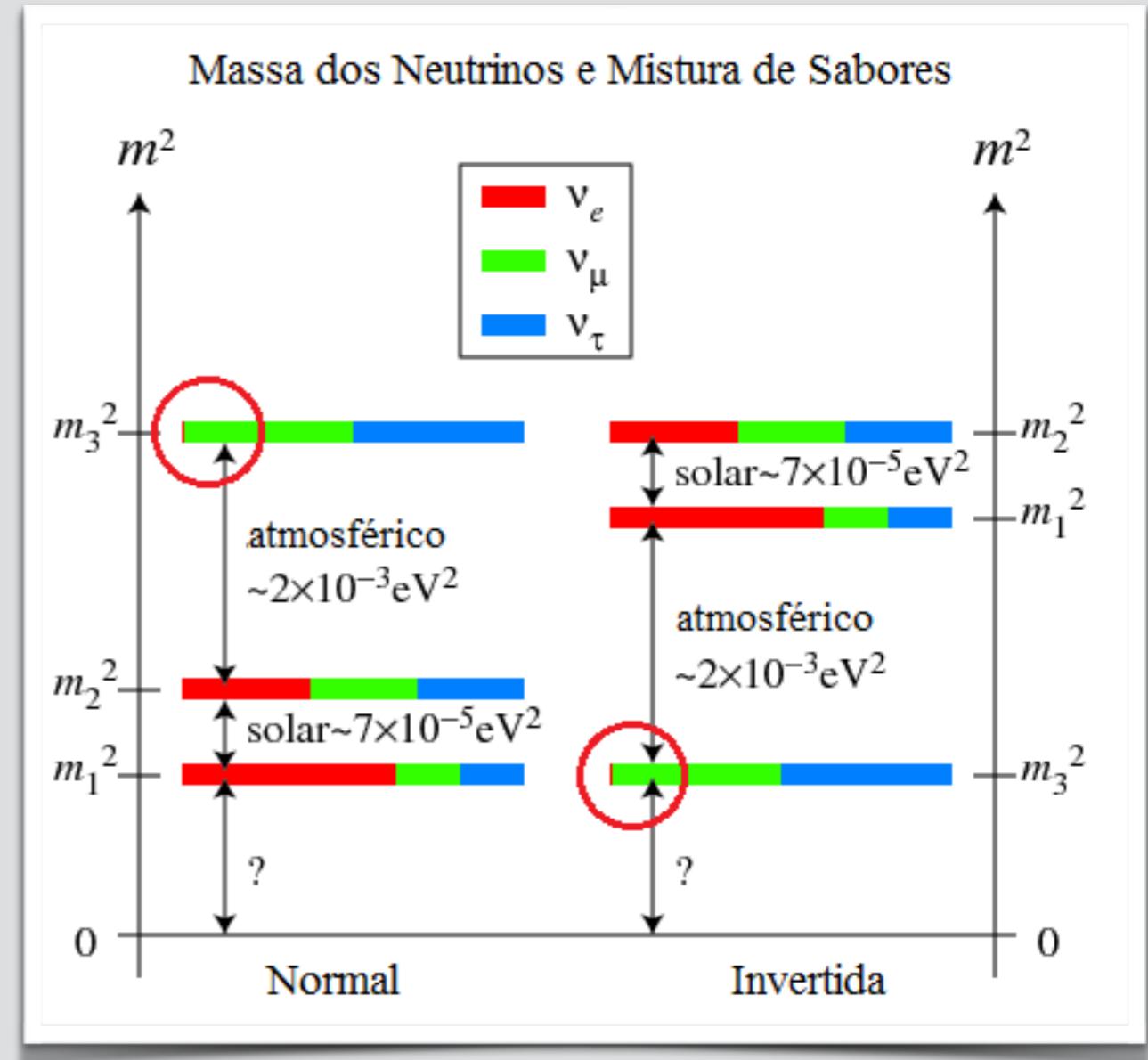
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$$\nu_e \xrightarrow{E, x} \nu_\tau$$

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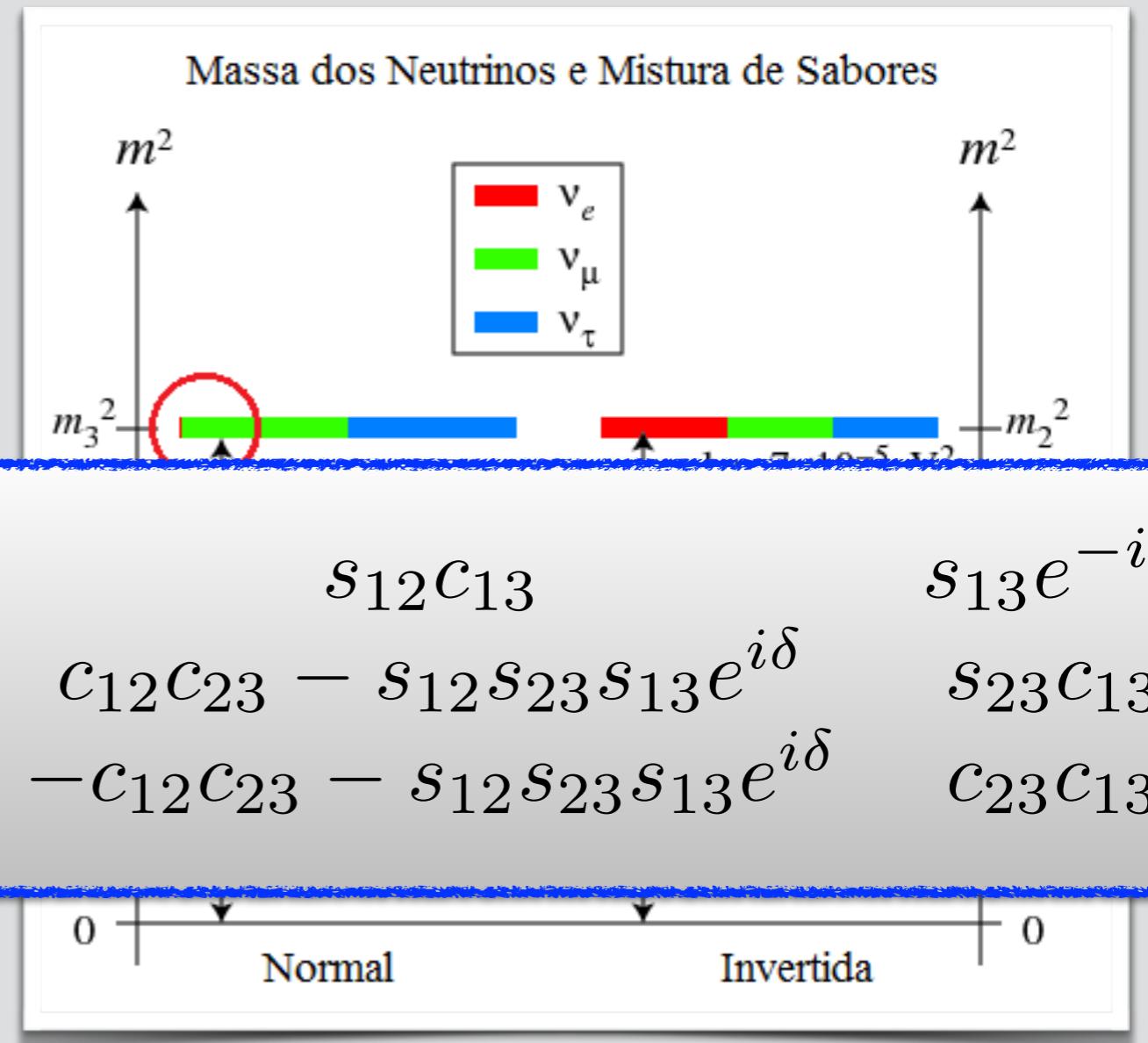
# Neutrino Oscillation

$$\nu_e \xrightarrow[E, x]{} \nu_\tau$$

$$U = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & -c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix}$$

$$|\nu_\alpha\rangle = \sum_i U_i^* |\nu_i\rangle$$

$\alpha = e, \mu, \tau$  and  $i = 1, 2, 3$



$$x_{\text{osc}} = \frac{4\pi E}{\Delta m_{ij}^2}$$

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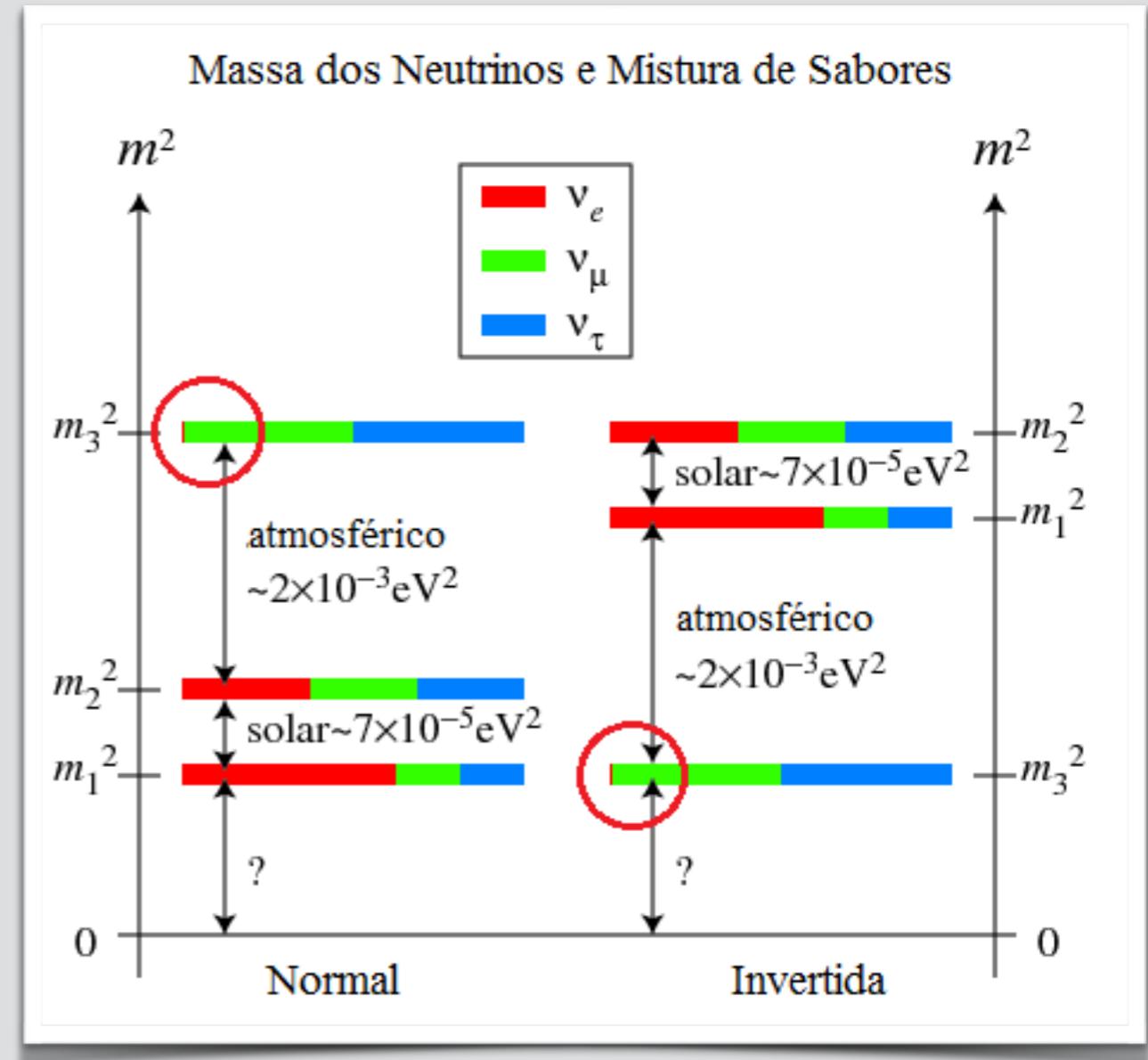
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$(\alpha = e, \mu, \tau \text{ and } i = 1, 2, 3)$



$$x_{\text{osc}} = \frac{4\pi E}{\Delta m_{ij}^2}$$

$$P(\nu_{\alpha} \rightarrow \nu_{\beta}) = \sum_{i,j} U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^* \exp \left( i \frac{m_j^2 - m_i^2}{2E} x \right)$$

# Neutrino Oscillation

Massa dos Neutrinos e Mistura de Sabores

$$\nu_e \xrightarrow[E, x]{} \nu_e$$

$$|\nu_i\rangle = \sum_{\alpha} U_{\alpha i} |\nu_{\alpha}\rangle$$

$$|\nu_{\alpha}\rangle = \sum_i U_{\alpha i}^* |\nu_i\rangle$$

## The Nobel Prize in Physics 2015



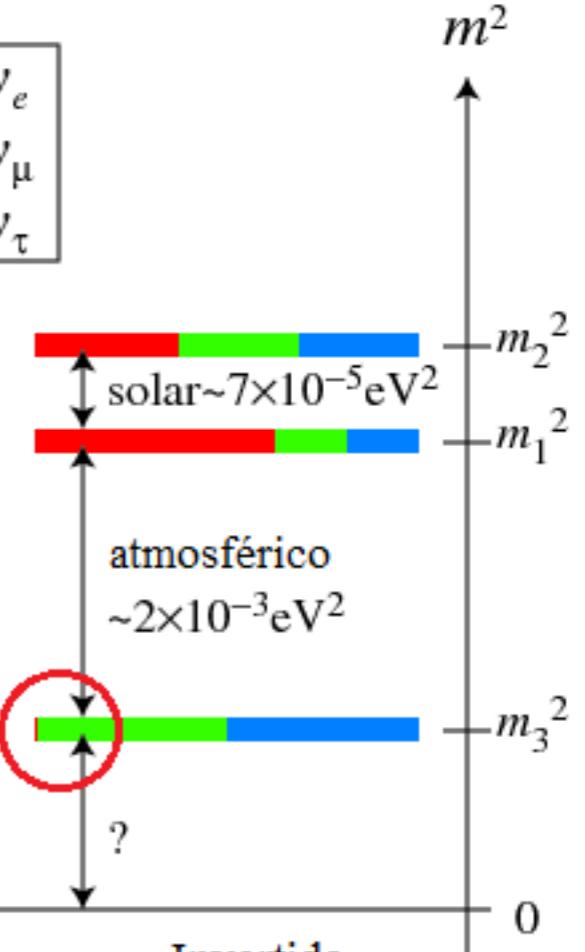
Photo: A. Mahmoud  
**Takaaki Kajita**  
Prize share: 1/2



Photo: A. Mahmoud  
**Arthur B. McDonald**  
Prize share: 1/2

The Nobel Prize in Physics 2015 was awarded jointly to Takaaki Kajita and Arthur B. McDonald "for the discovery of neutrino oscillations, which shows that neutrinos have mass"

$\nu_e$   
 $\nu_{\mu}$   
 $\nu_{\tau}$



$$x_{\text{osc}} = \frac{4\pi E}{\Delta m_{ij}^2}$$

$$P(\nu_{\alpha} \rightarrow \nu_{\beta}) = \sum_{i,j} U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^* \exp \left( i \frac{m_j^2 - m_i^2}{2E} x \right)$$

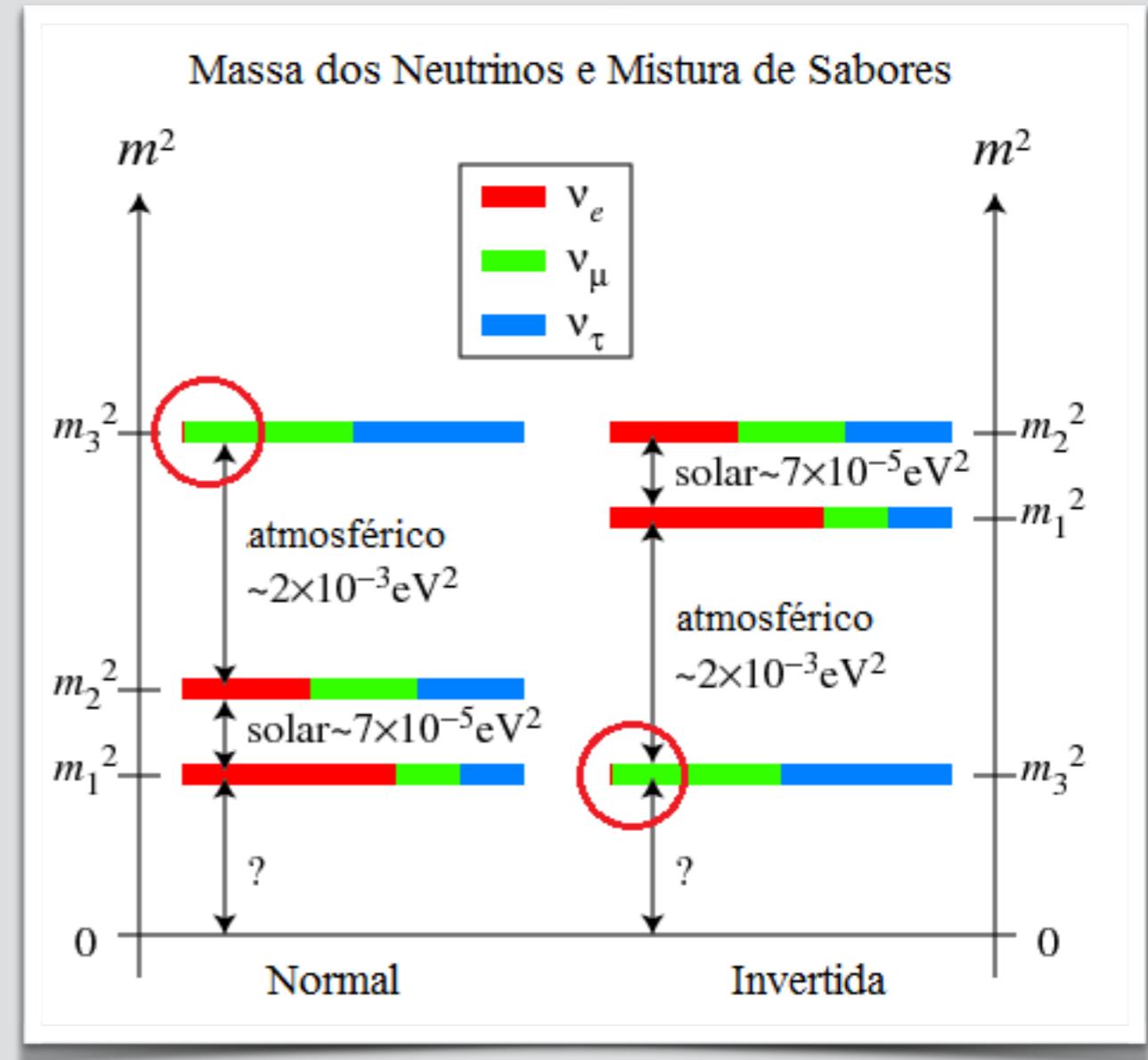
# Neutrino Oscillation

$$\nu_e \xrightarrow{E, x} \nu_\tau$$

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$i$  ( $\alpha = e, \mu, \tau$  and  $i = 1, 2, 3$ )



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# Neutrino Oscillation

<http://pdg.lbl.gov/2017/reviews/rpp2016-rev-neutrino-mixing.pdf>

**Table 14.4:** Sensitivity of different oscillation experiments.

Source	Type of $\nu$	$\bar{E}[\text{MeV}]$	$L[\text{km}]$	$\min(\Delta m^2)[\text{eV}^2]$
Reactor	$\bar{\nu}_e$	$\sim 1$	1	$\sim 10^{-3}$
Reactor	$\bar{\nu}_e$	$\sim 1$	100	$\sim 10^{-5}$
Accelerator	$\nu_\mu, \bar{\nu}_\mu$	$\sim 10^3$	1	$\sim 1$
Accelerator	$\nu_\mu, \bar{\nu}_\mu$	$\sim 10^3$	1000	$\sim 10^{-3}$
Atmospheric $\nu$ 's	$\nu_{\mu,e}, \bar{\nu}_{\mu,e}$	$\sim 10^3$	$10^4$	$\sim 10^{-4}$
Sun	$\nu_e$	$\sim 1$	$1.5 \times 10^8$	$\sim 10^{-11}$

$$|\nu_\alpha\rangle = \sum_i U_i^* |\nu_i\rangle$$

$i$  ( $\alpha = e, \mu, \tau$  and  $i = 1, 2, 3$ )



$$x_{\text{osc}} = \frac{4\pi E}{\Delta m_{ij}^2}$$

$$P(\nu_\alpha \rightarrow \nu_\beta) = \sum_{i,j} U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^* \exp\left(i \frac{m_j^2 - m_i^2}{2E} x\right)$$

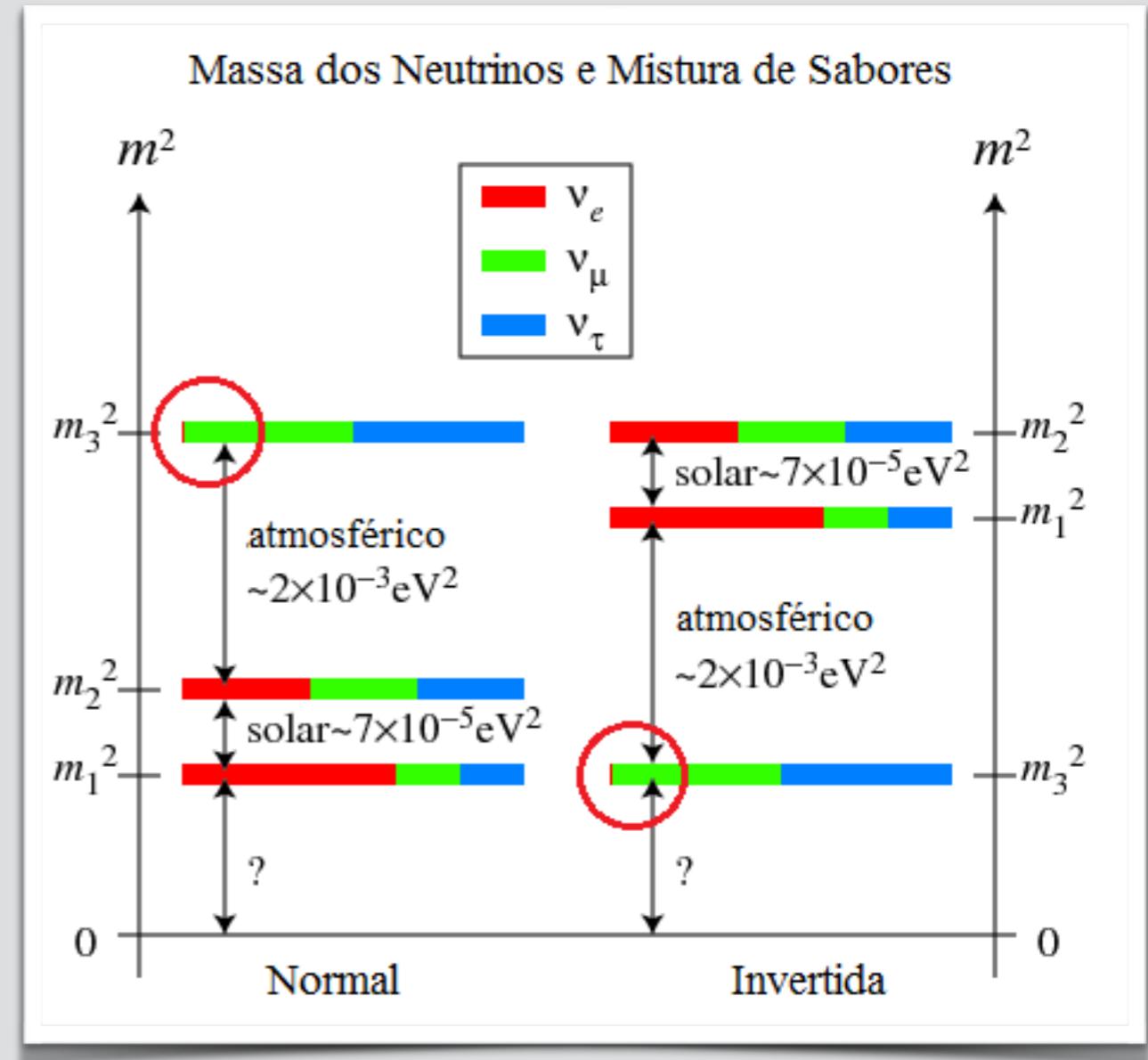
# Neutrino Oscillation

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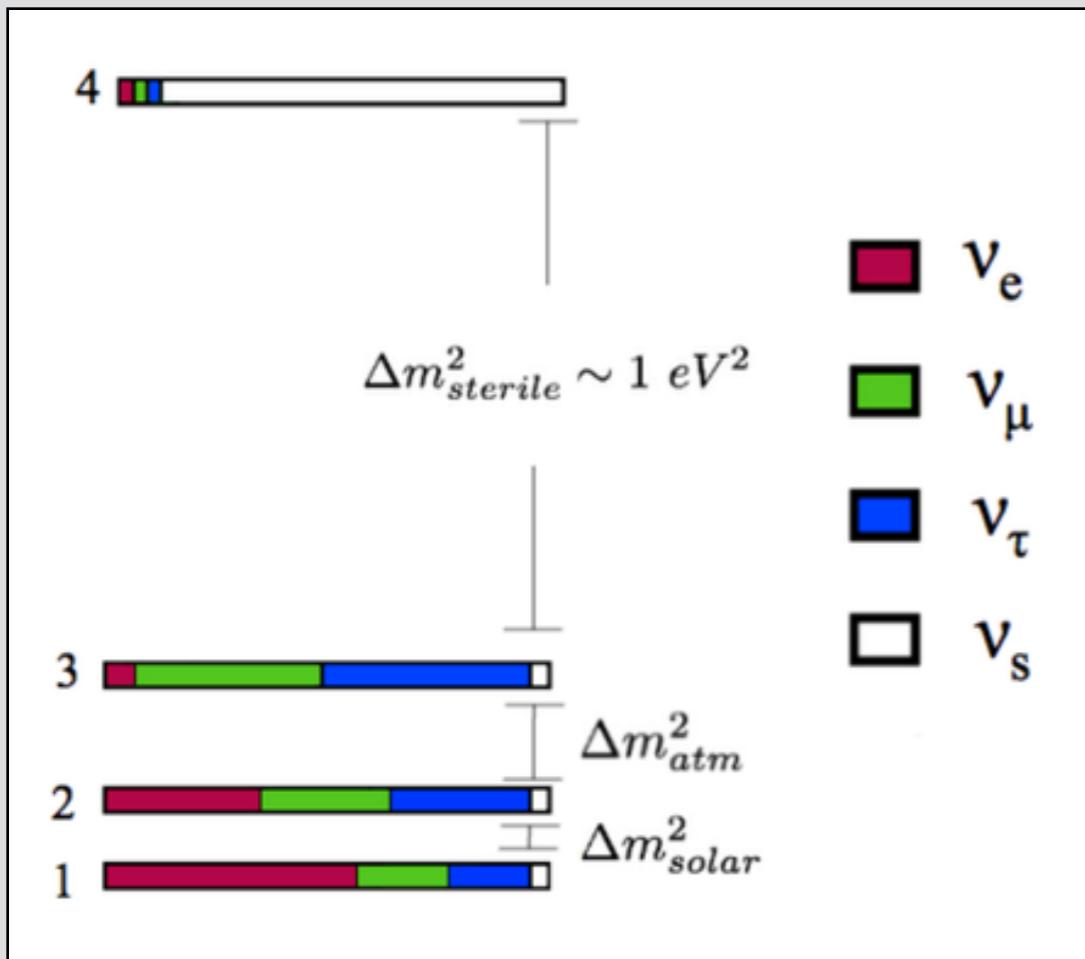
$i$  ( $\alpha = e, \mu, \tau$  and  $i = 1, 2, 3$ )



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# 3+1 model



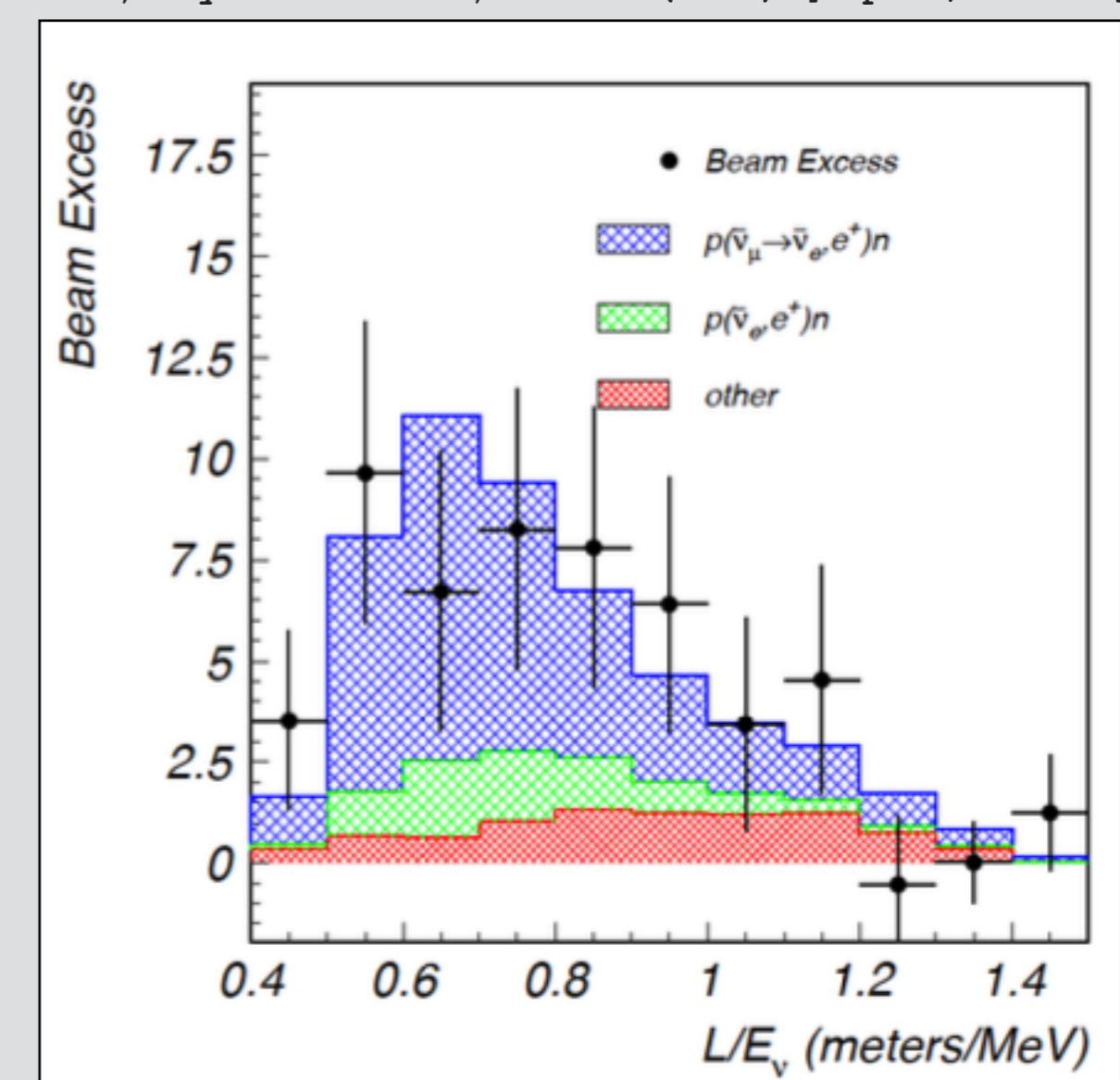
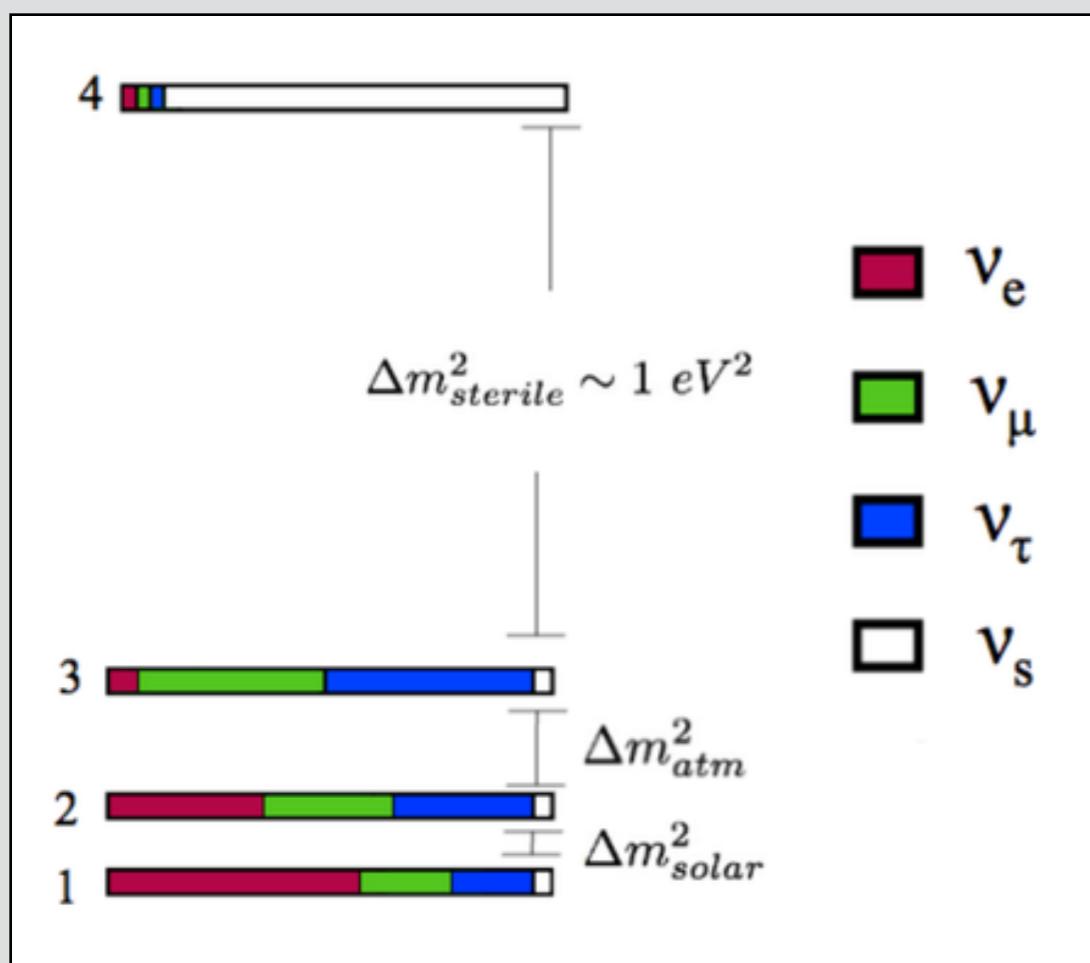
$$U_{3+1} = \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} \\ \vdots & & \vdots & U_{\mu 4} \\ \vdots & & \vdots & U_{\tau 4} \\ U_{s1} & U_{s2} & U_{s3} & U_{s4} \end{bmatrix}$$

$$P(\nu_\mu \rightarrow \nu_\mu) = 1 - \sin^2(2\theta_{\mu\mu}) \sin^2(1.27\Delta m_{41}^2 L/E)$$

$$P(\nu_\mu \rightarrow \nu_e) = \sin^2(2\theta_{\mu e}) \sin^2(1.27\Delta m_{41}^2 L/E)$$

$x_{\text{osc}} \approx 1 \text{ km}$

# 3+1 model

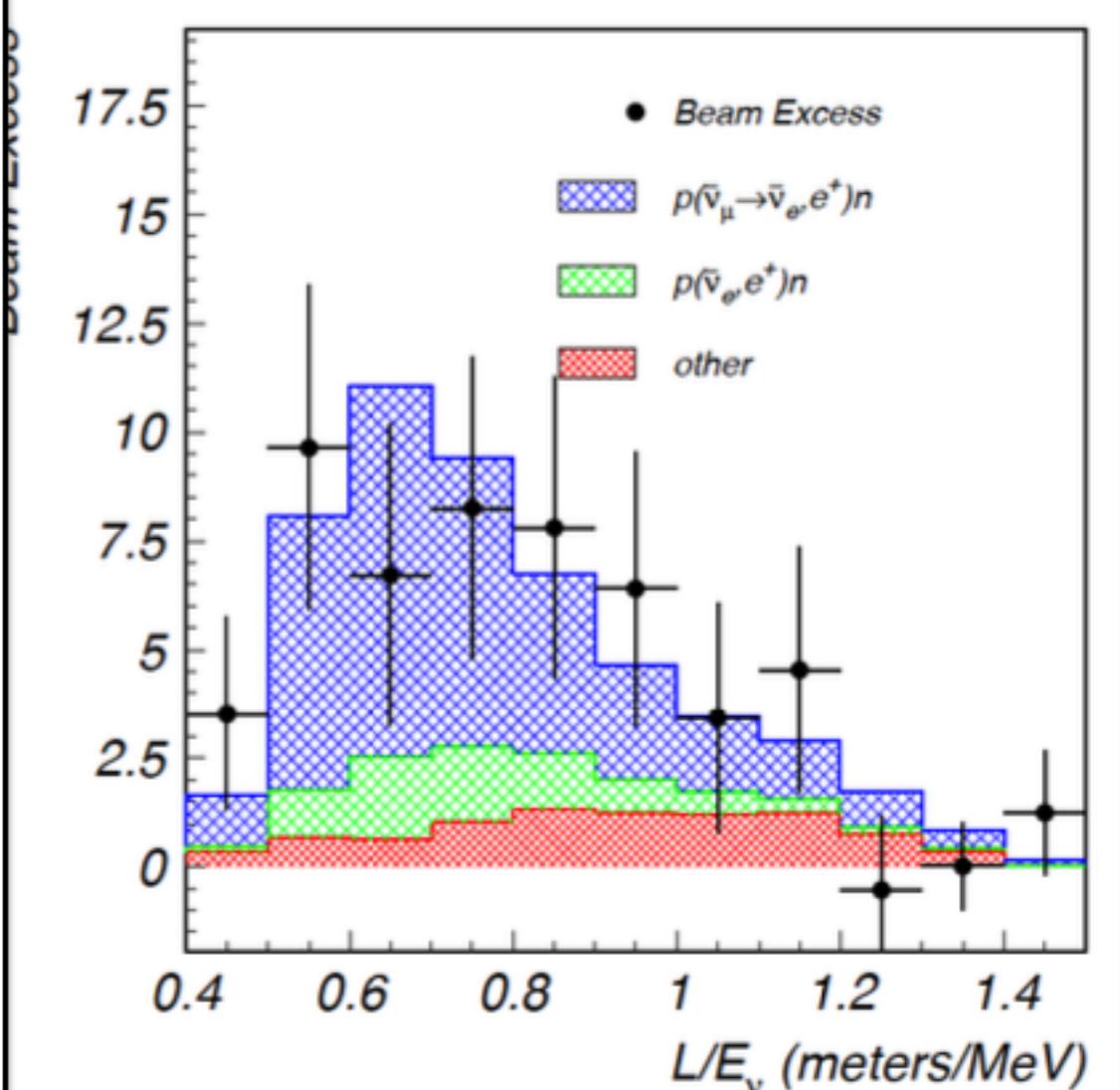
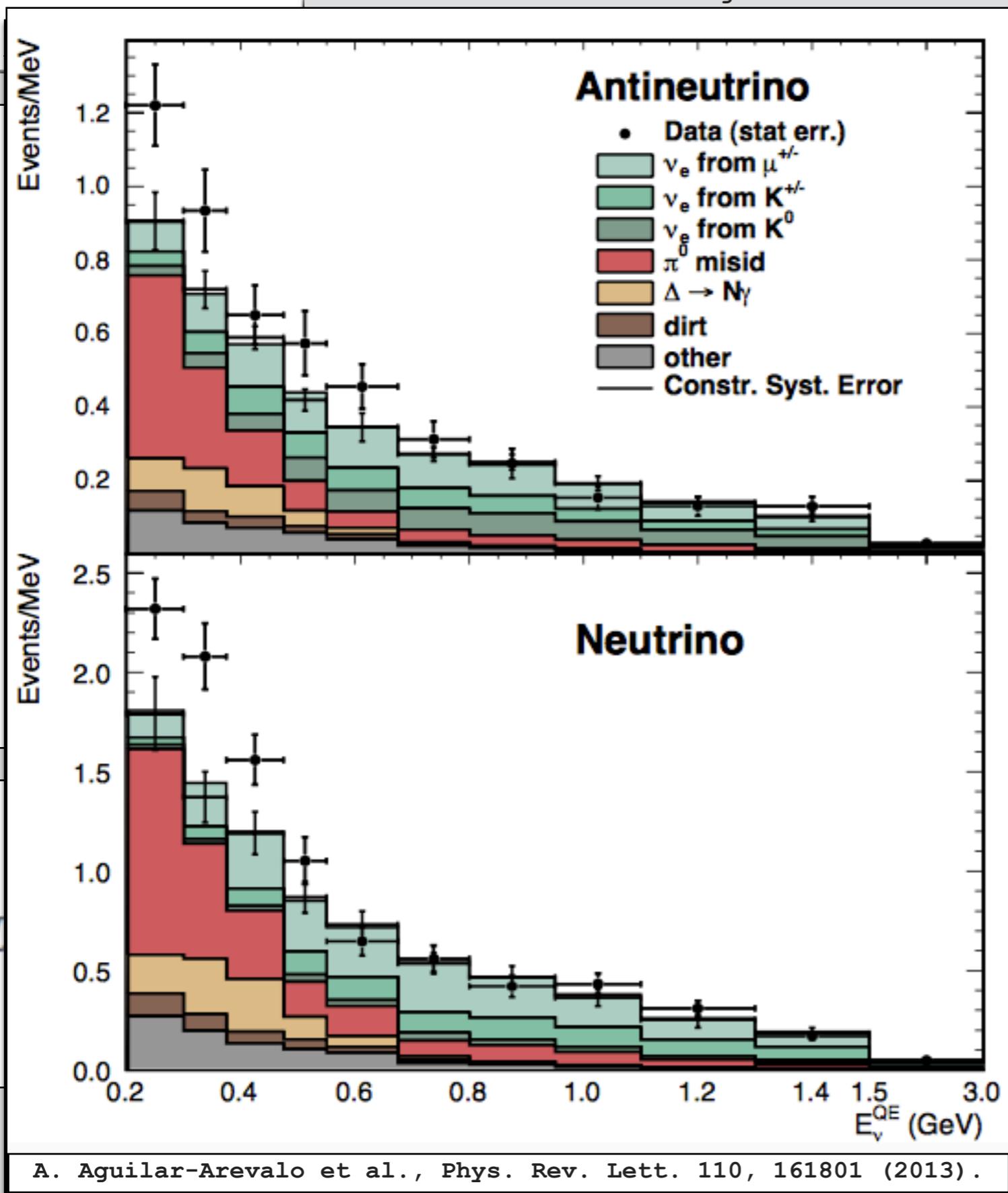


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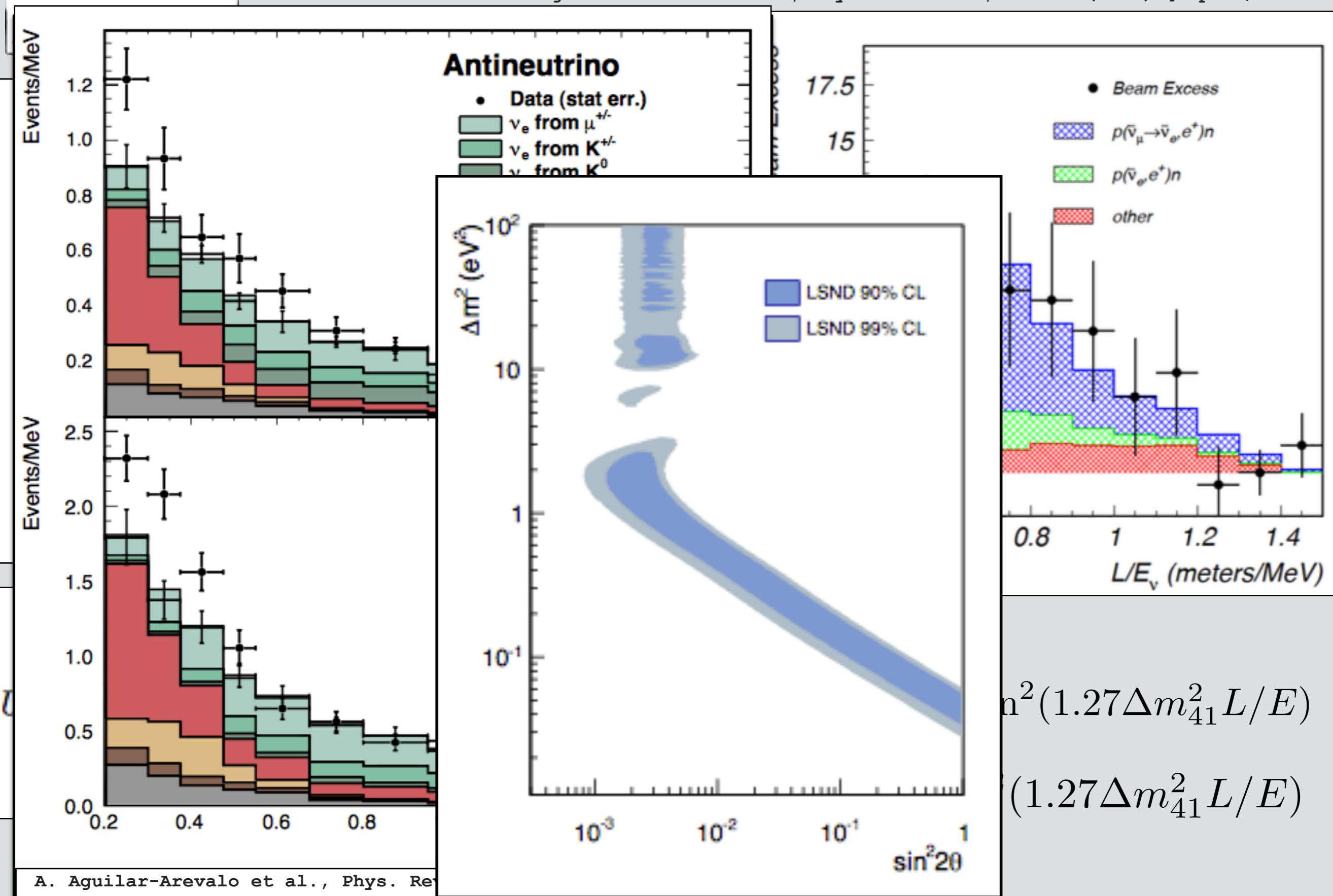


$$\sin^2(2\theta_{\mu\mu}) \sin^2(1.27\Delta m_{41}^2 L/E)$$

$$\sin^2(2\theta_{\mu e}) \sin^2(1.27\Delta m_{41}^2 L/E)$$

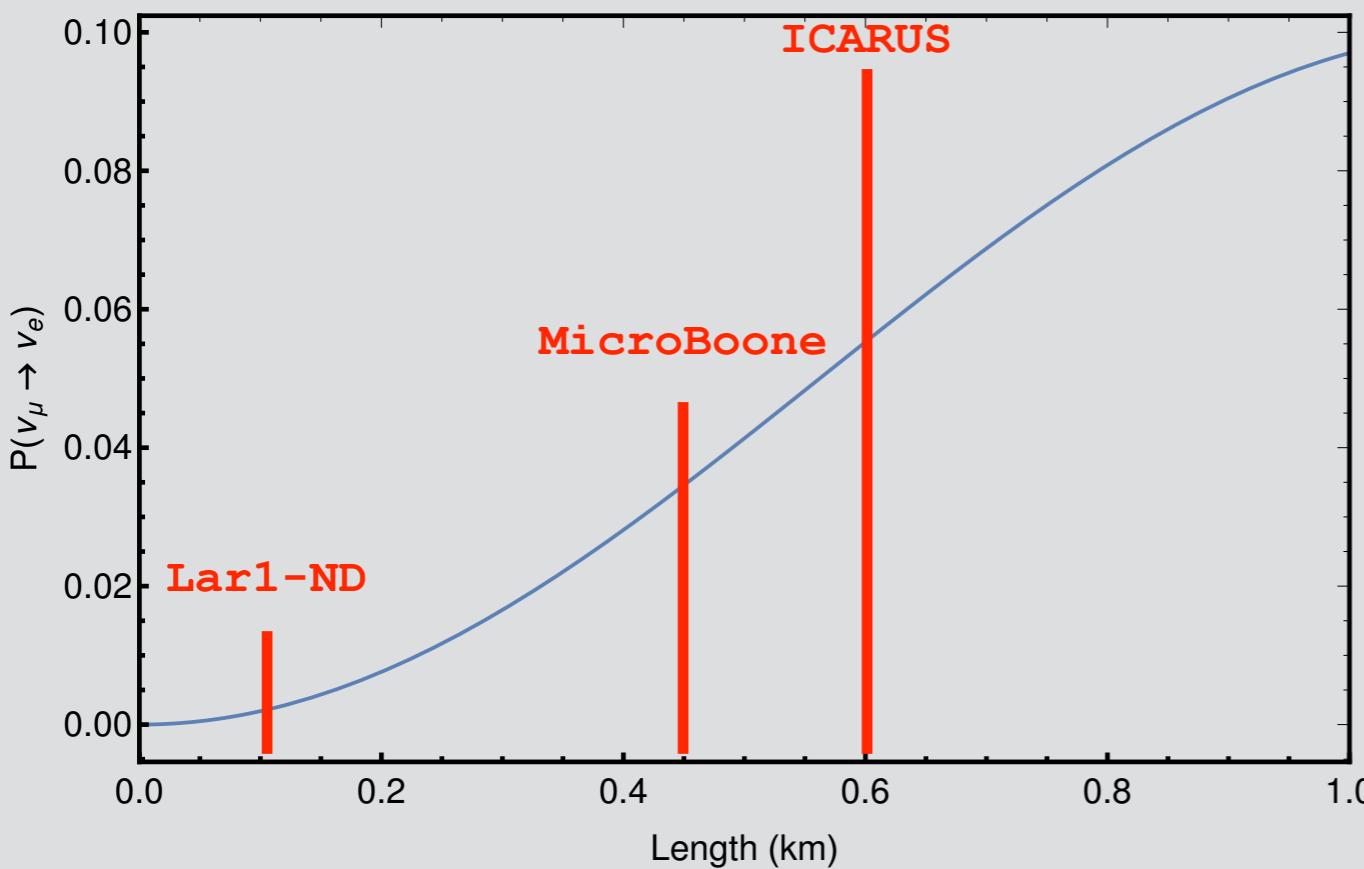
A. Aguilar-Arevalo et al., Phys. Rev. Lett. 110, 161801 (2013).

$x_{\text{osc}} \approx 1\text{km}$

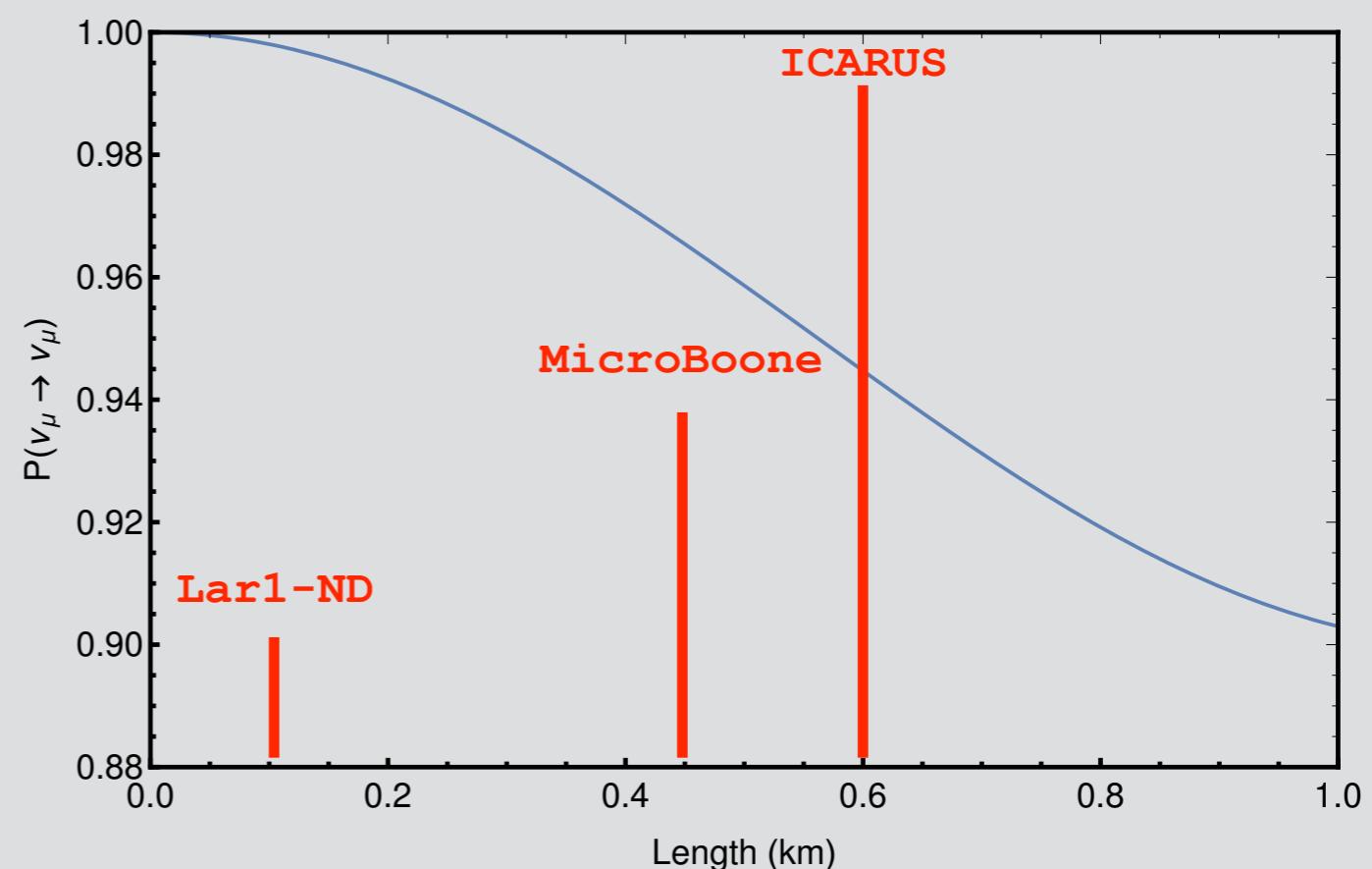


$x_{\text{osc}} \approx 1\text{km}$

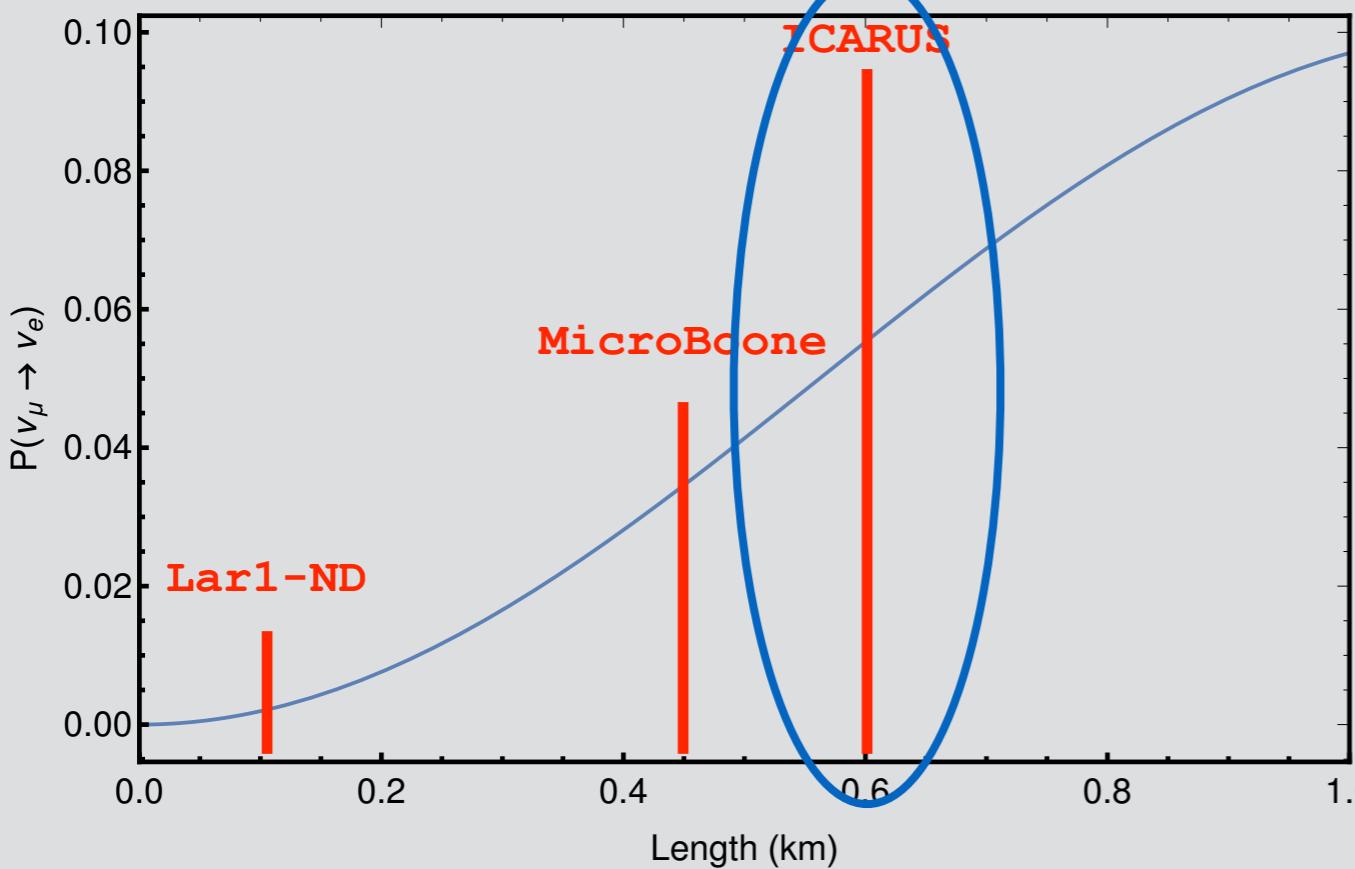
# 3+1 model at the SBNF



$$\sin^2(2\theta_{\mu\mu}) = 0.1$$
$$\sin^2(2\theta_{\mu e}) = 0.1$$
$$\Delta m_{41}^2 = 1.1 \text{ eV}^2$$



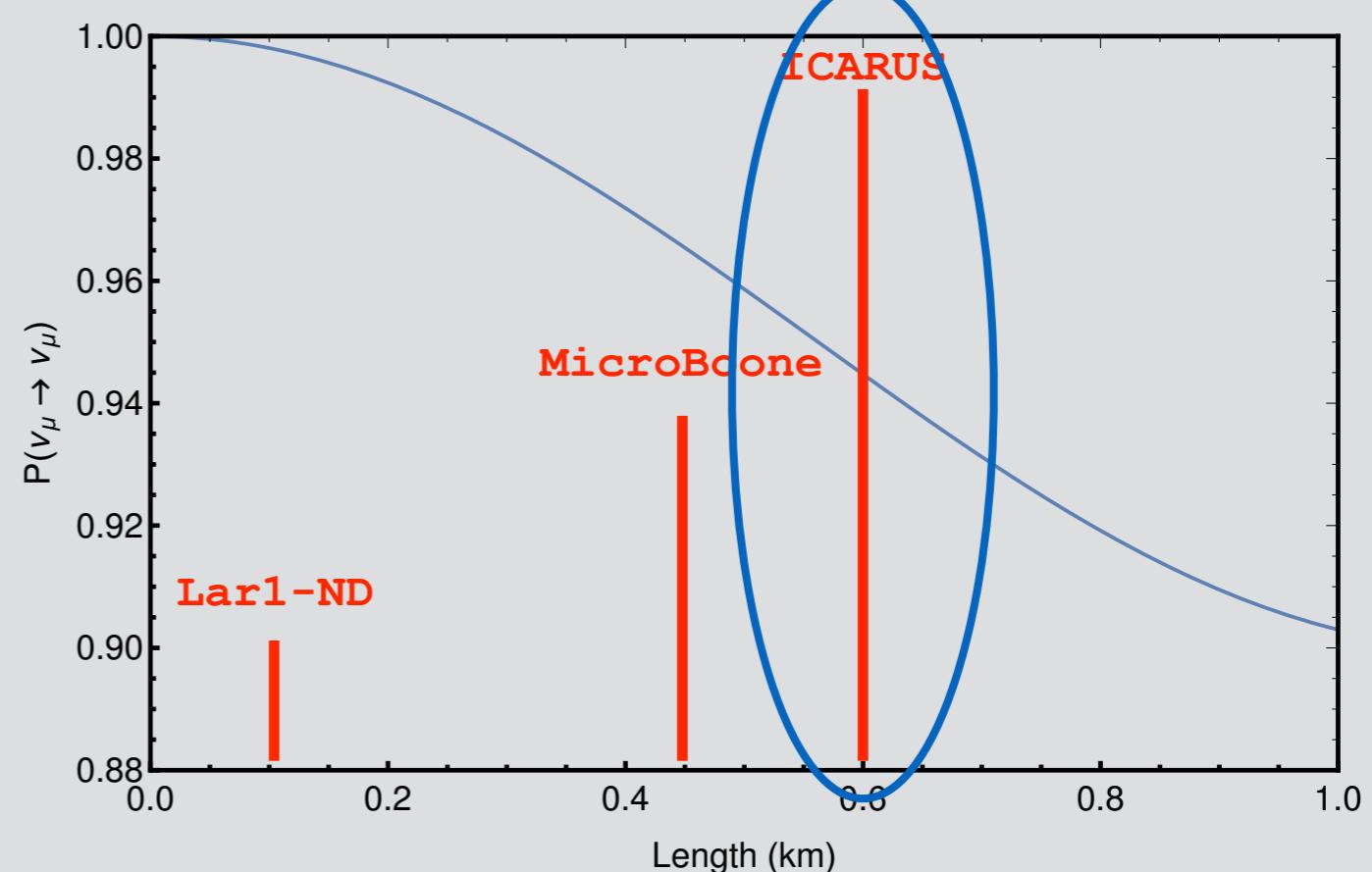
# 3+1 model at the SBNF



$$\sin^2(2\theta_{\mu\mu}) = 0.1$$

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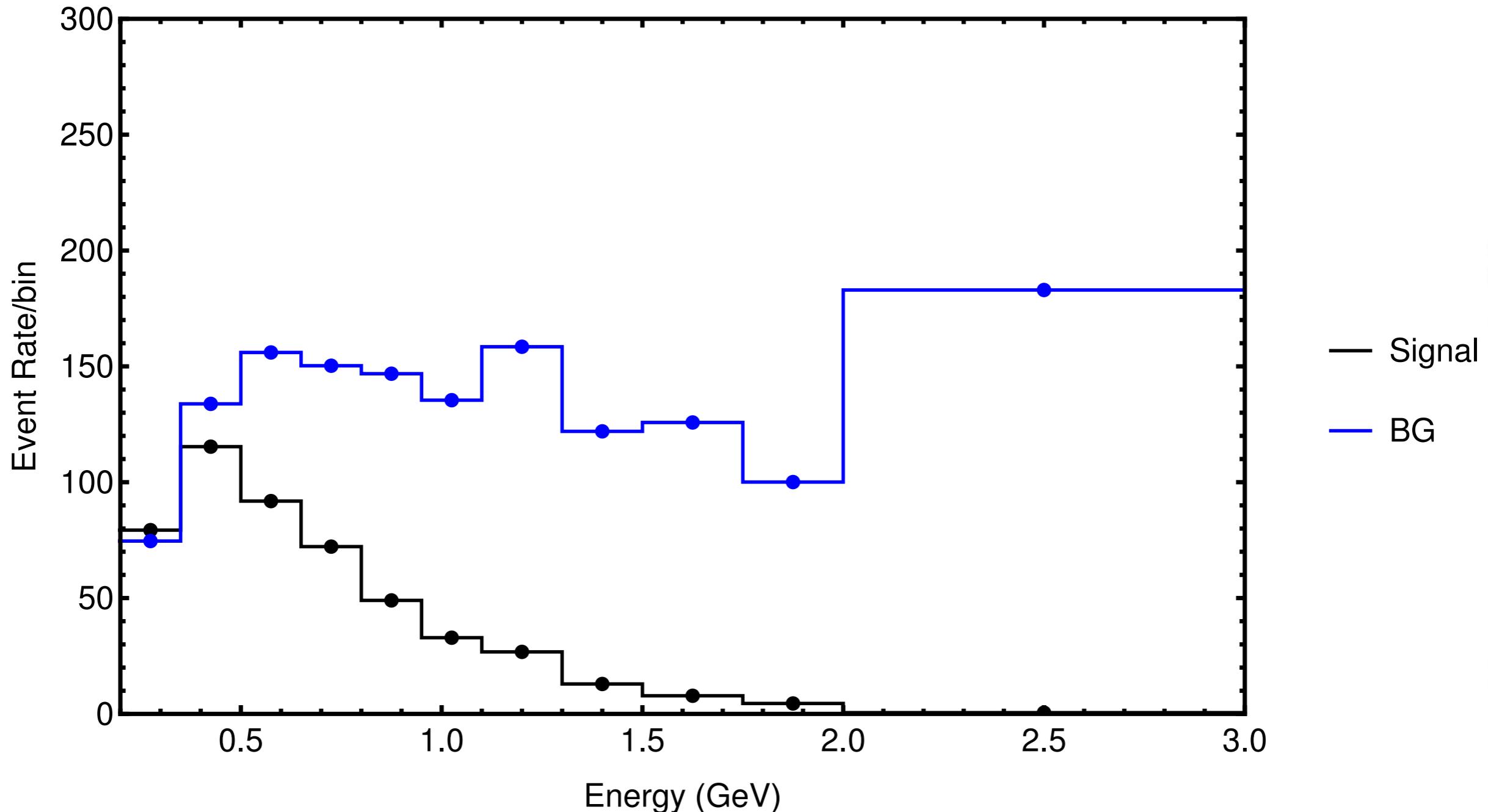
# 3+1 model at the SBNF

$\nu_e$

ICARUS,  $6.6 \times 10^{20}$  POT (600m)

Signal : ( $\Delta m_{41}^2 = 0.43 \text{ eV}^2$ ,  $\sin^2 2\theta_{\mu e} = 0.013$ )

Statistical Uncertainty Only



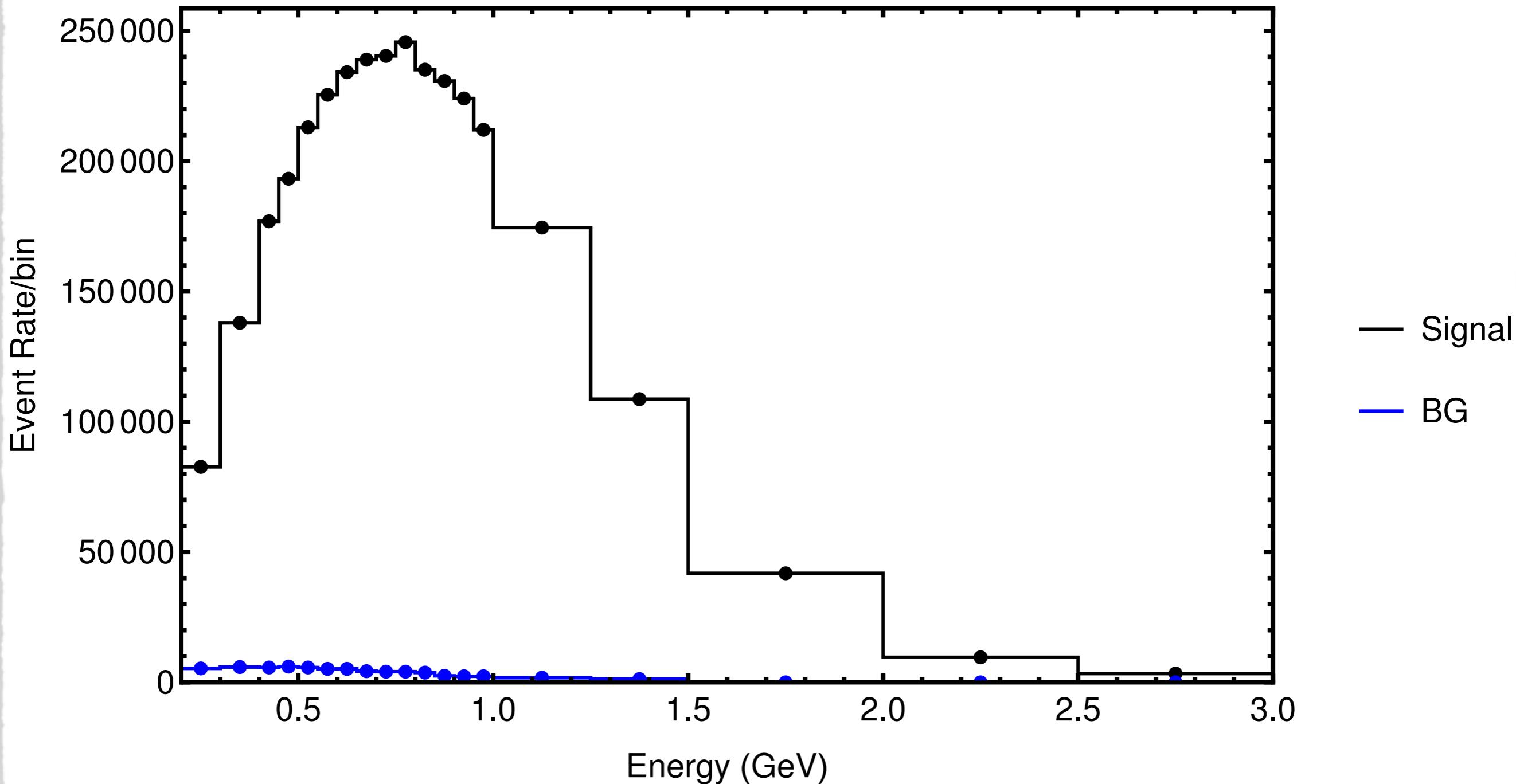
# 3+1 model at the SBNF

$\nu_\mu$

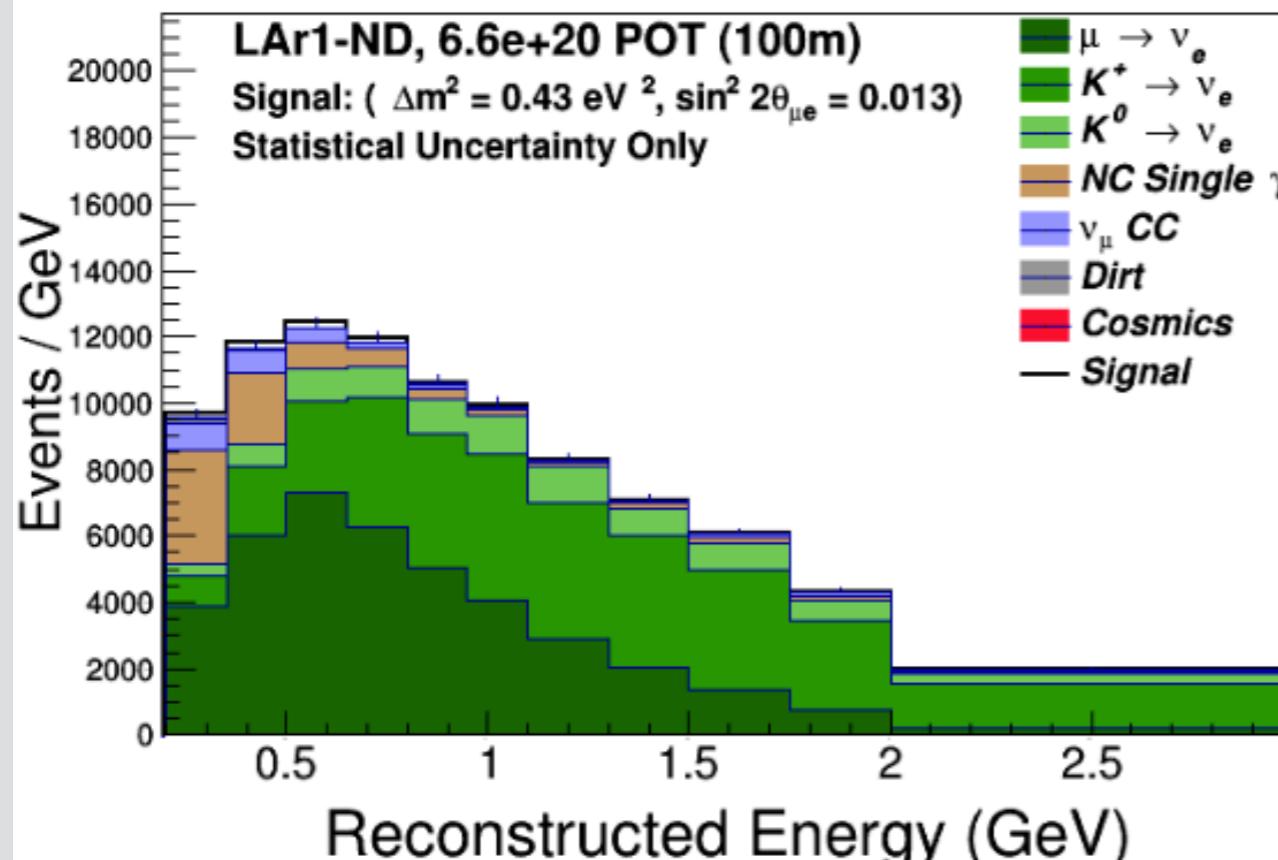
ICARUS,  $6.6 \times 10^{20}$  POT (600m)

Signal : ( $\Delta m_{41}^2 = 1.1 \text{ eV}^2$ ,  $\sin^2 2\theta_{\mu\mu} = 0.1$ )

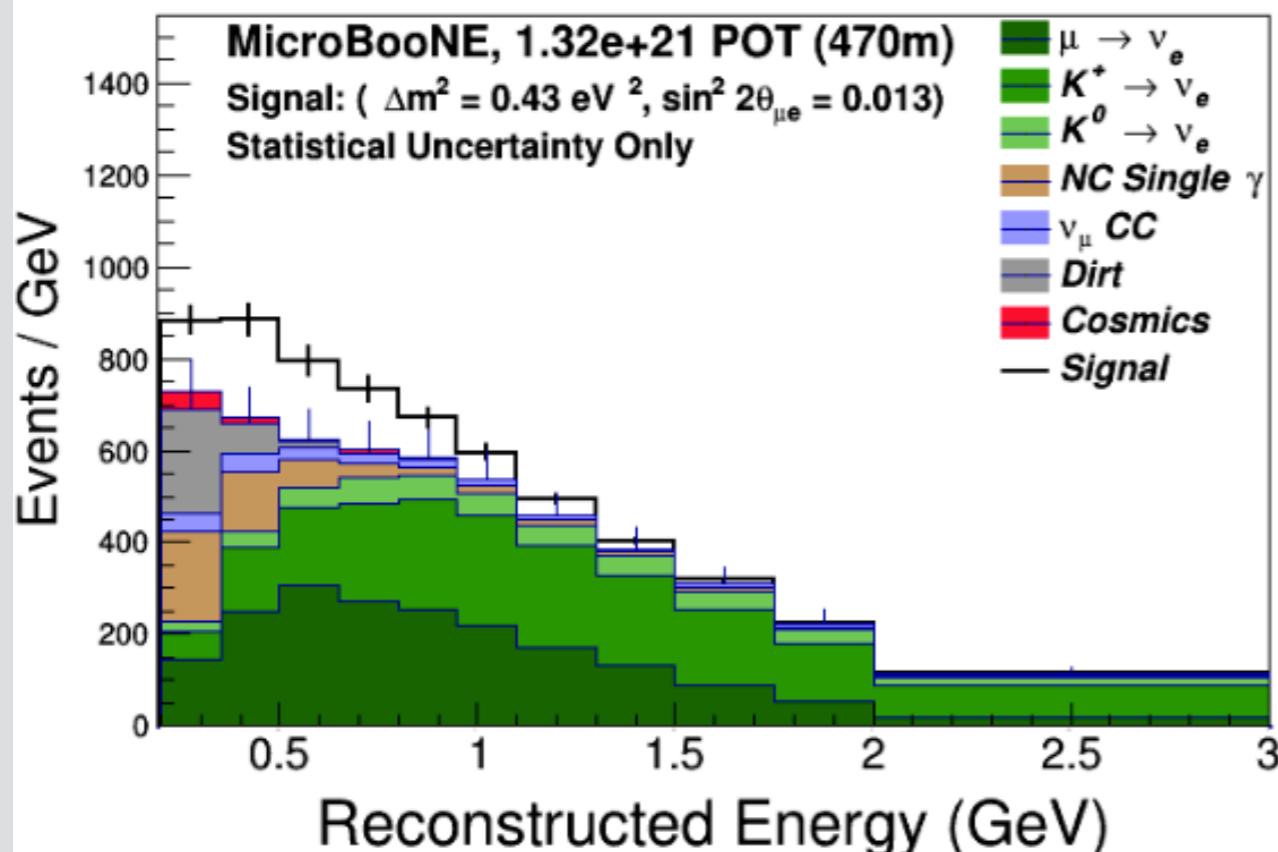
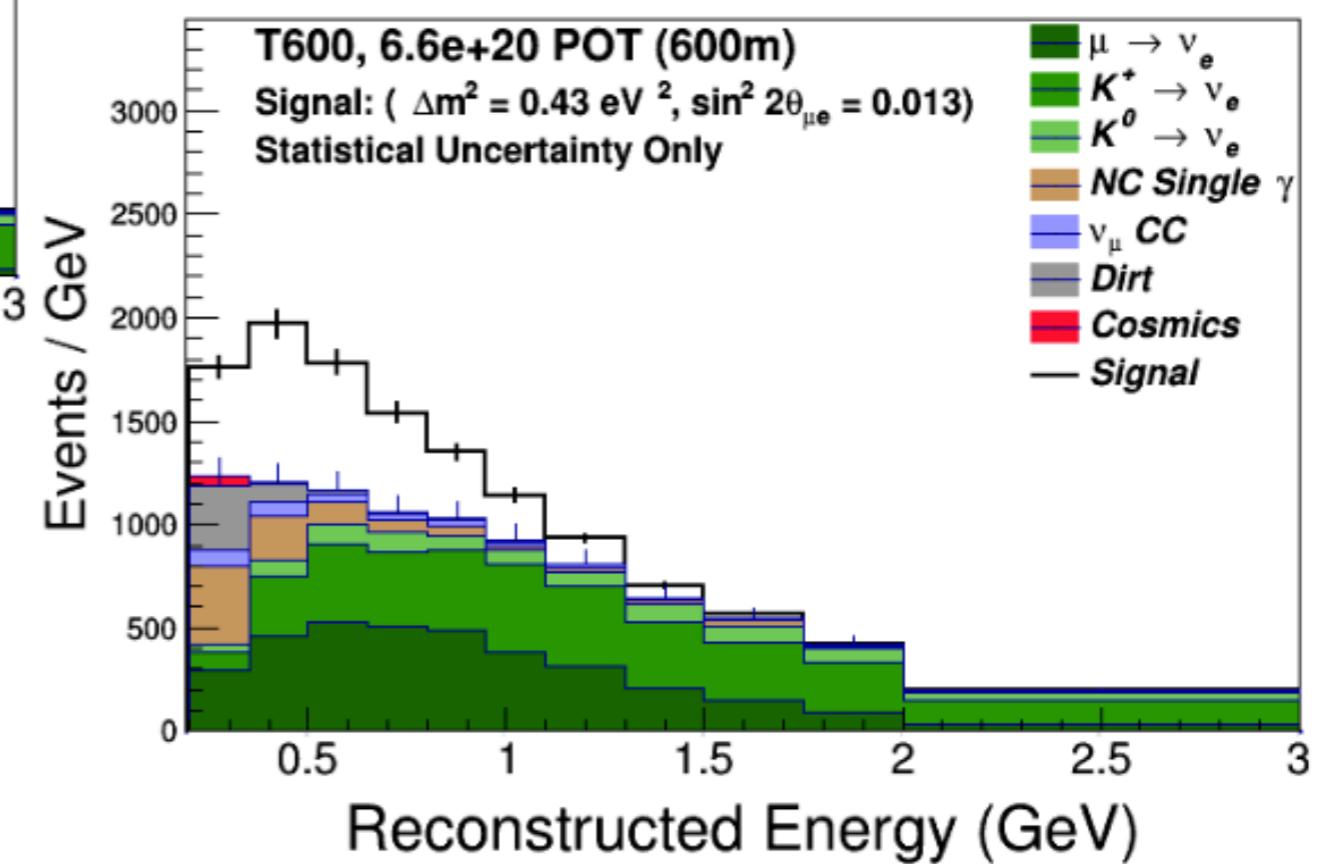
Statistical Uncertainty Only



# 3+1 model at the SBNF



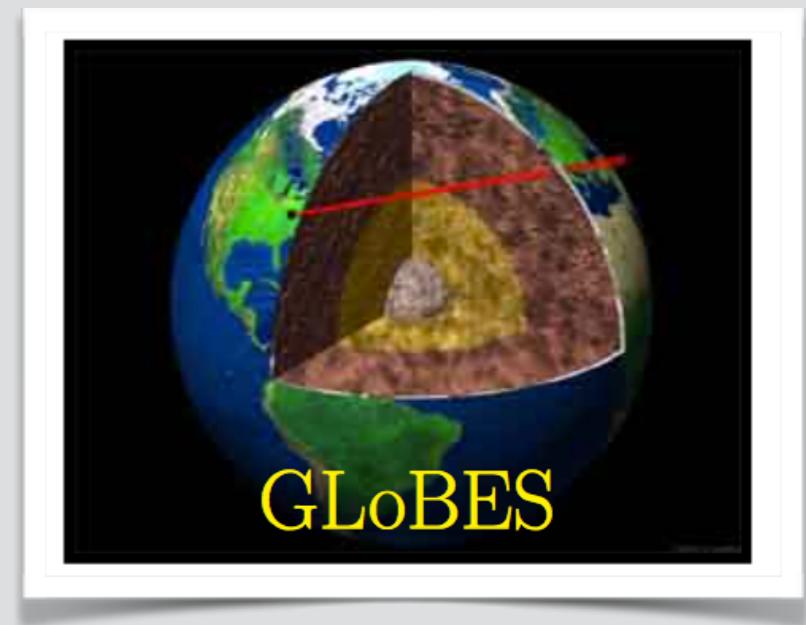
arXiv:1503.01520

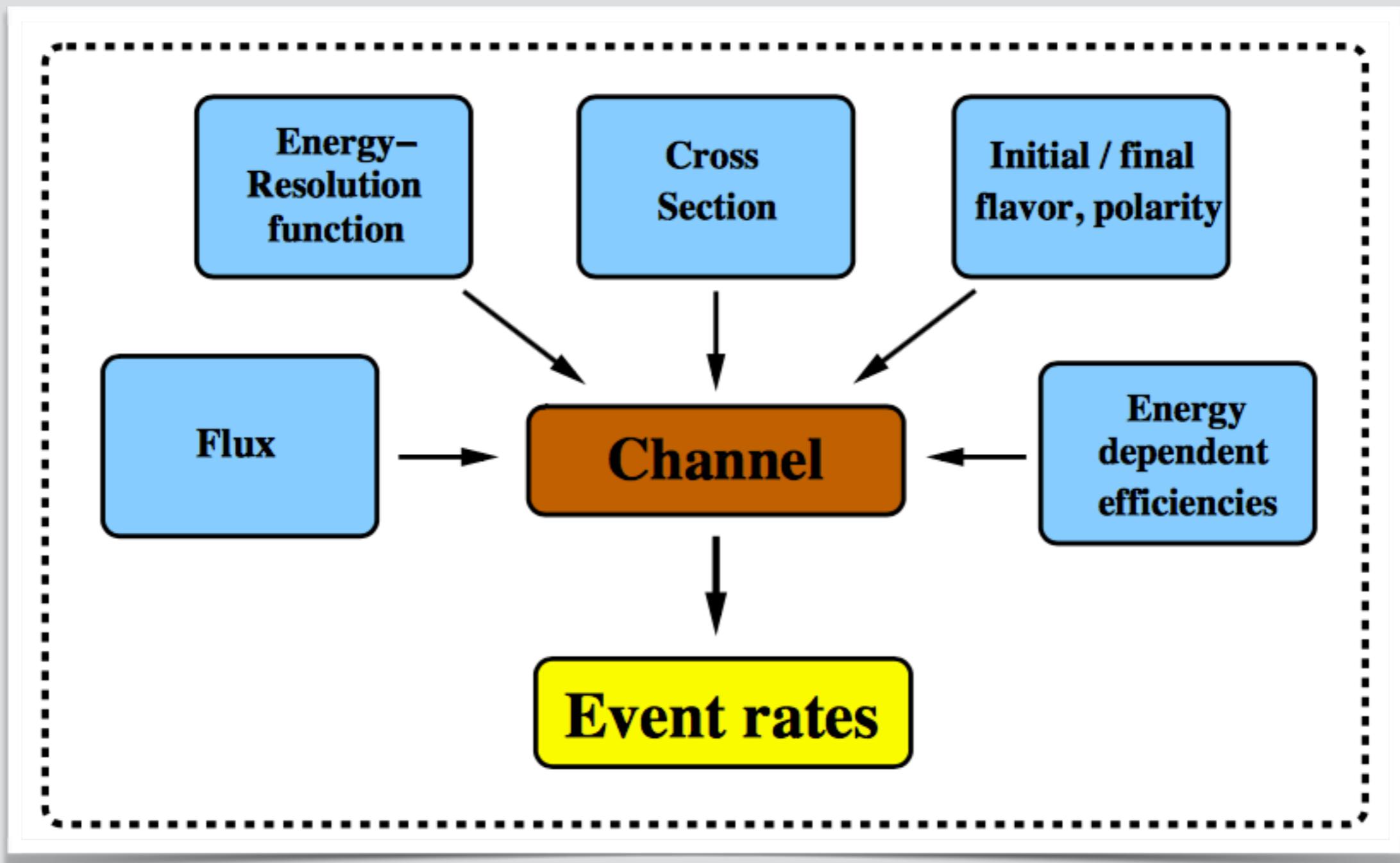


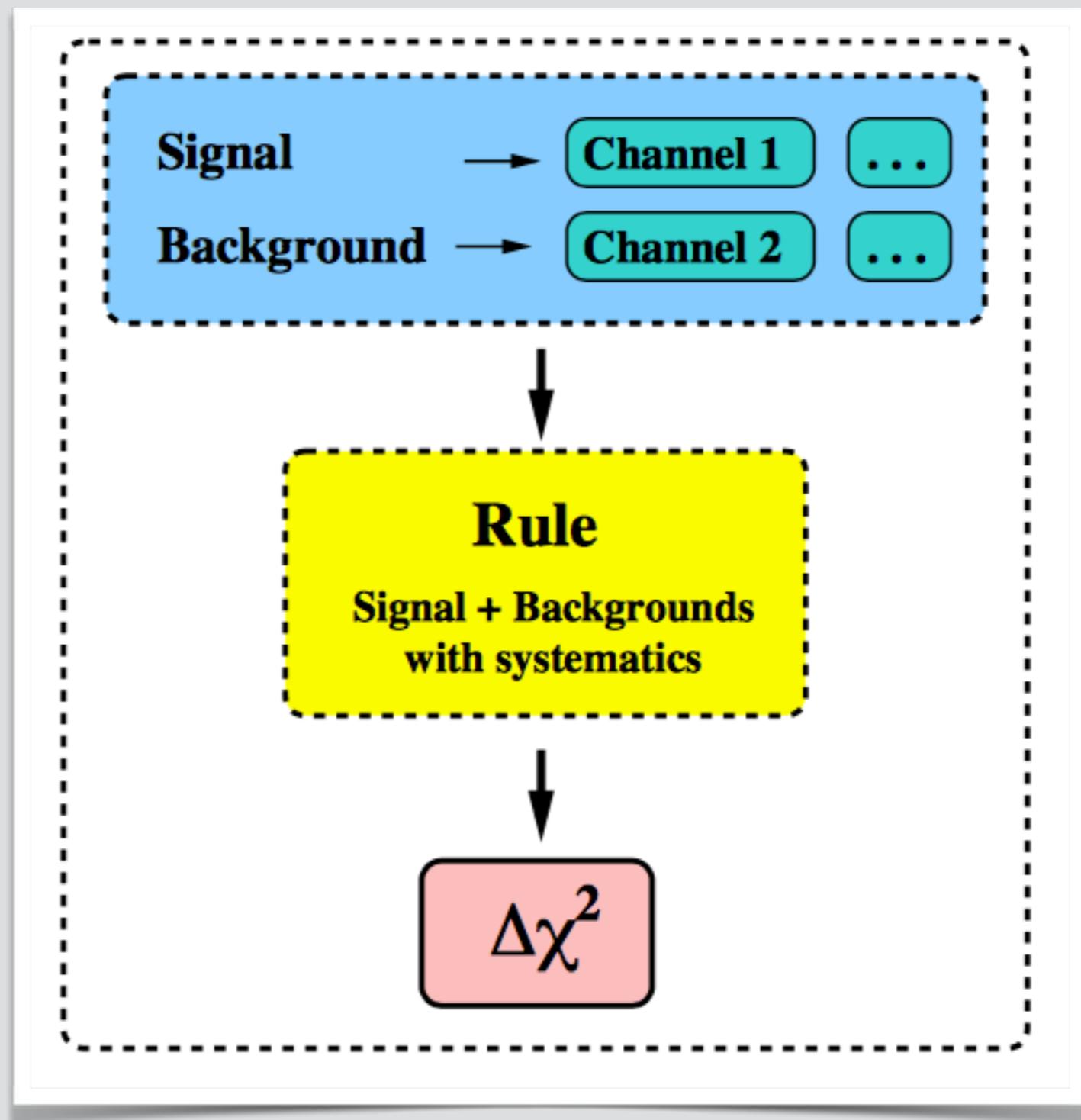
## General Long Baseline Experiment Simulator

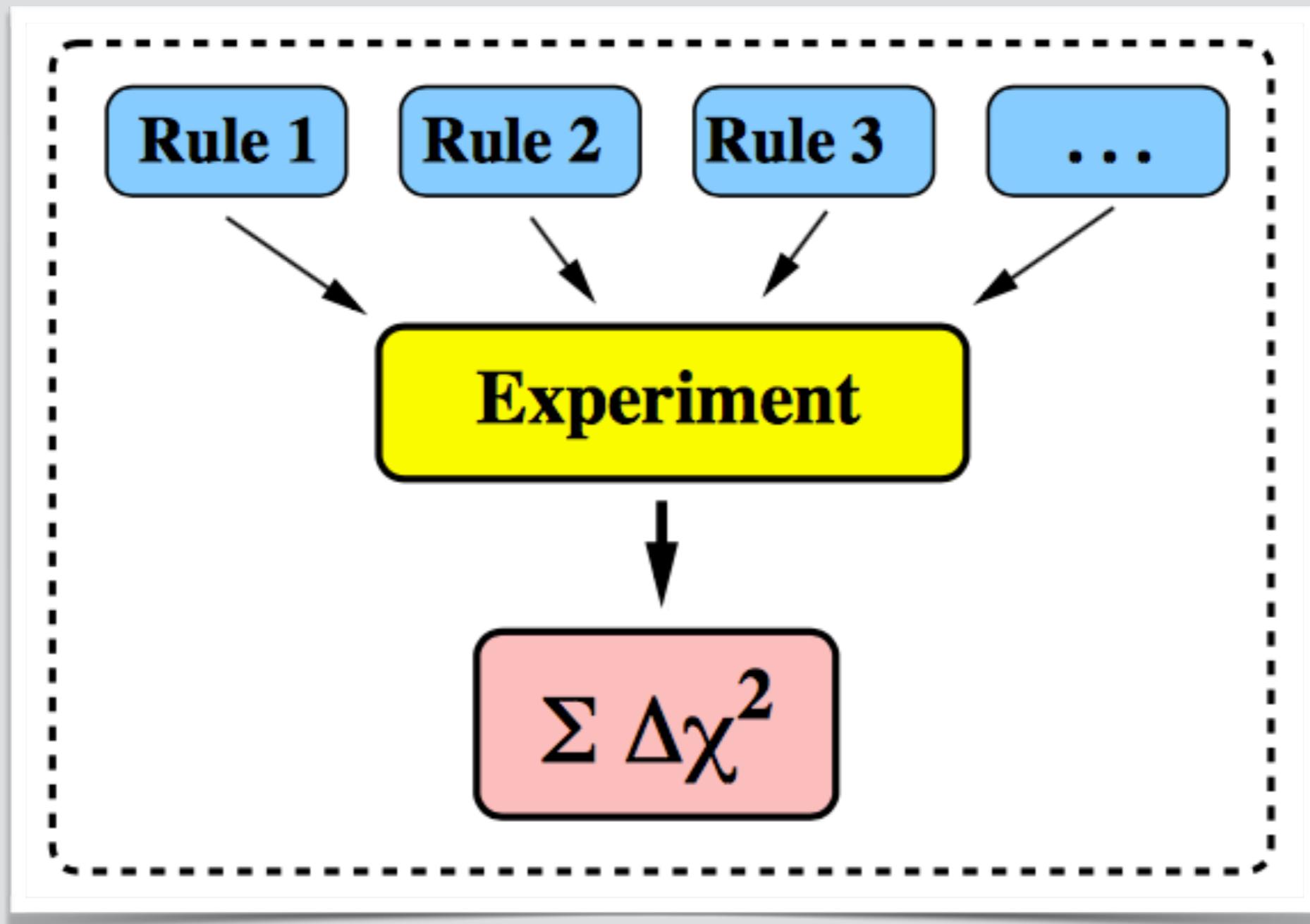


$$\frac{dn_{\beta}^{\text{IT}}}{dE'} = N \int_0^{\infty} \int_0^{\infty} dE d\hat{E} \underbrace{\Phi_{\alpha}(E)}_{\text{Production}} \times \underbrace{\frac{1}{L^2} P_{(\alpha \rightarrow \beta)}(E, L, \rho; \theta_{12}, \theta_{13}, \theta_{23}, \Delta m_{31}^2, \Delta m_{21}^2, \delta_{\text{CP}})}_{\text{Propagation}} \times \underbrace{\sigma_f^{\text{IT}}(E) k_f^{\text{IT}}(E - \hat{E})}_{\text{Interaction}} \times \underbrace{T_f(\hat{E}) V_f(\hat{E} - E')}_{\text{Detection}},$$

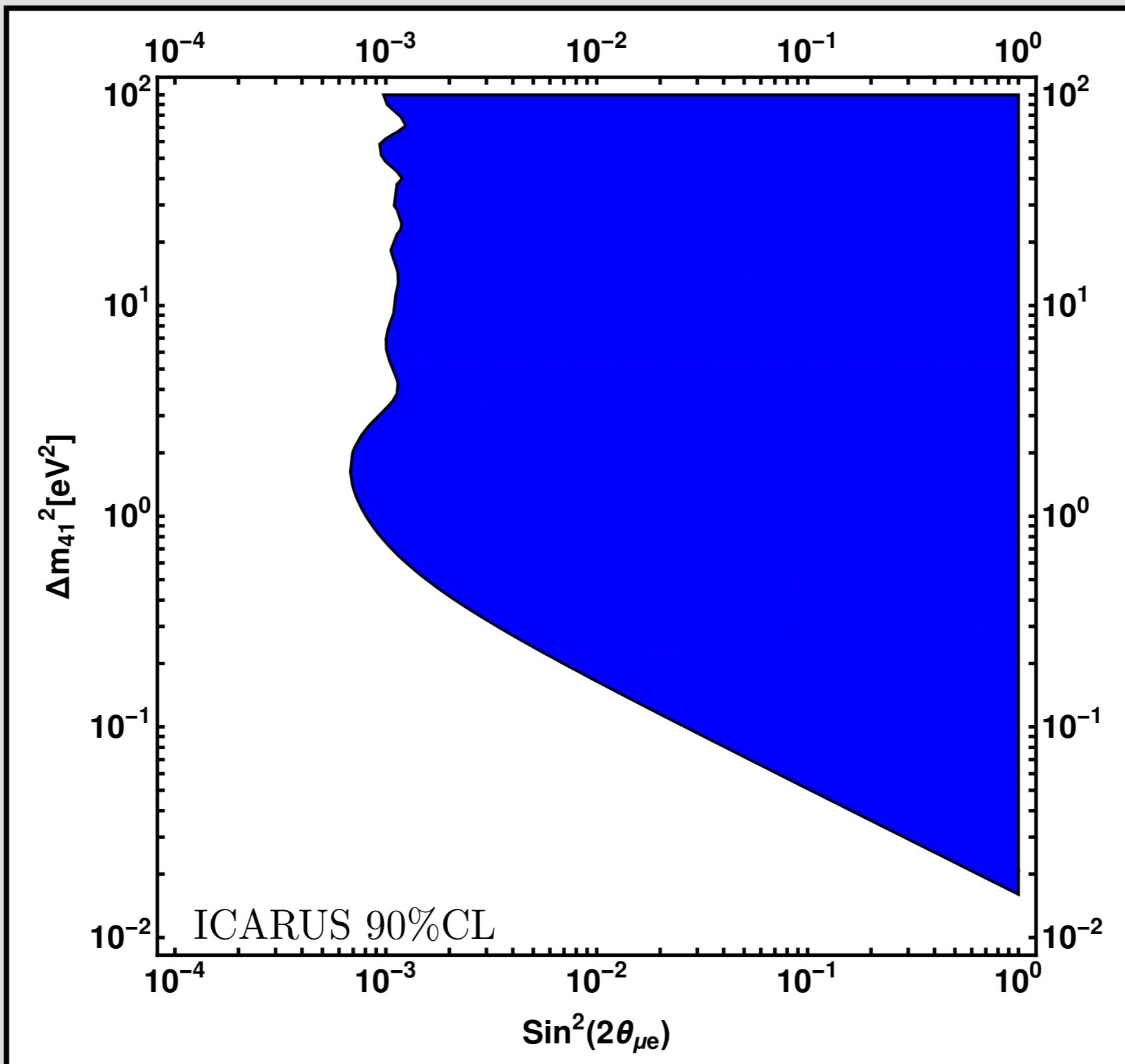






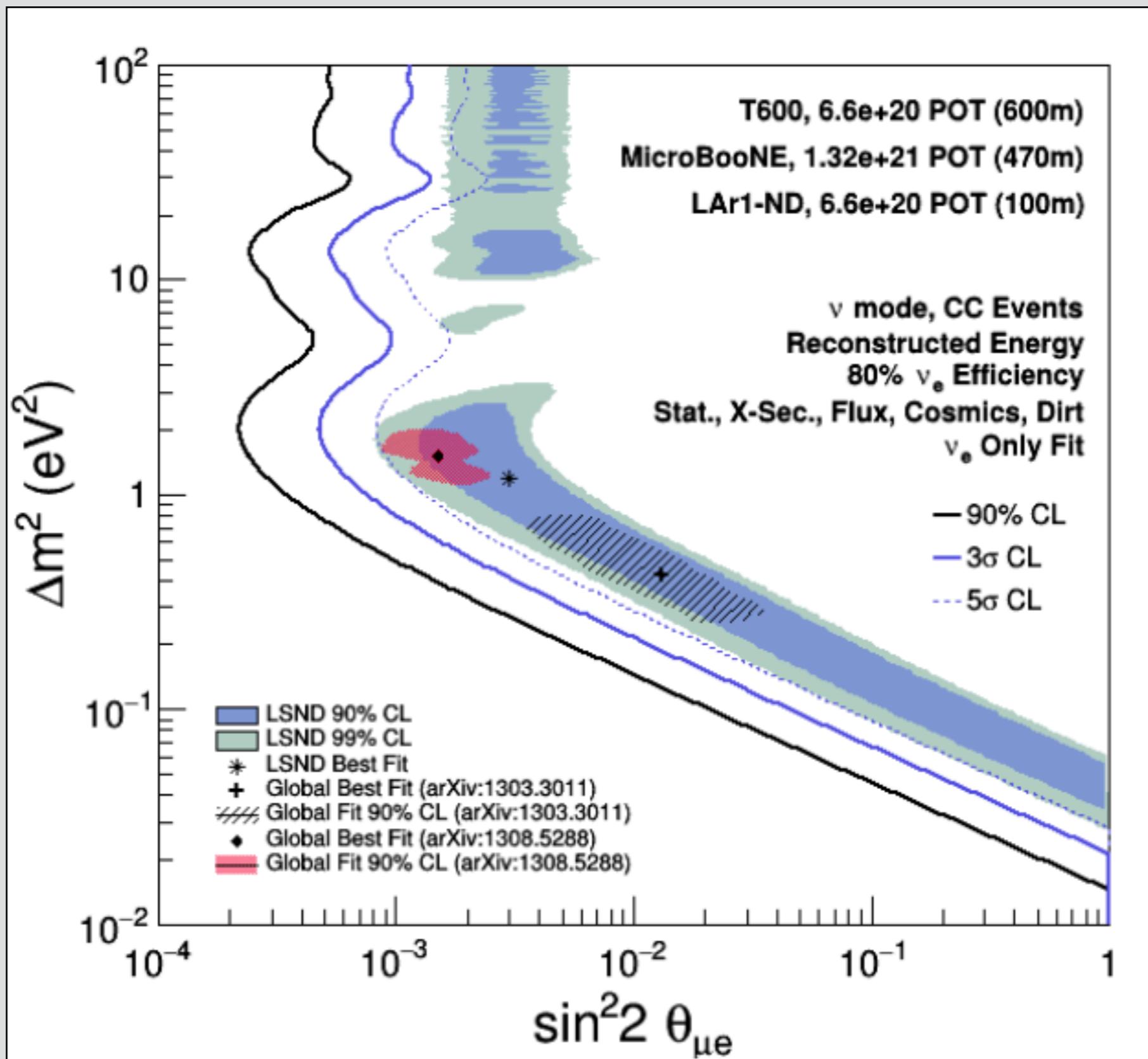


# 3+1 model at the SBNF

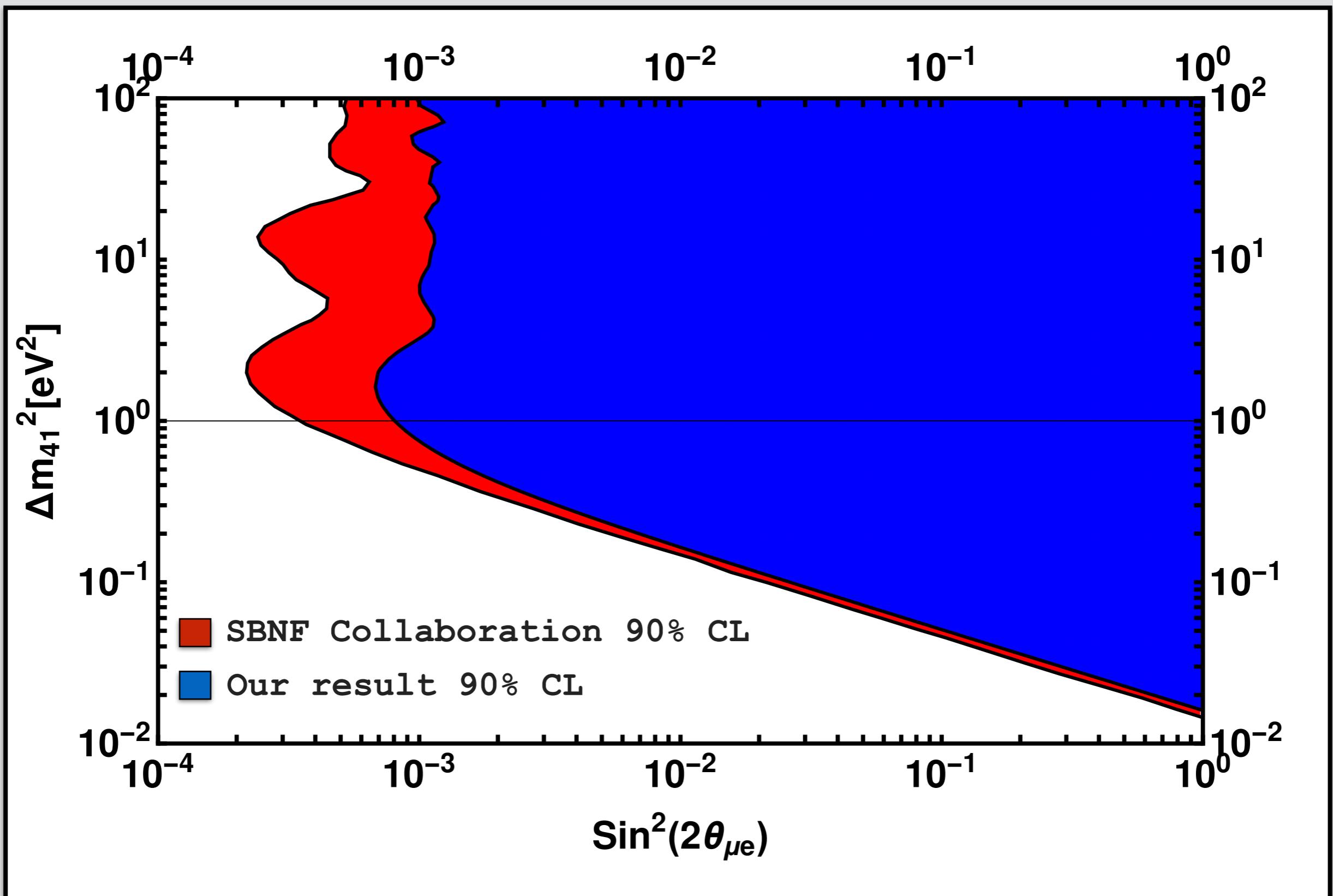


# 3+1 model at the SBNF

arXiv:1503.01520

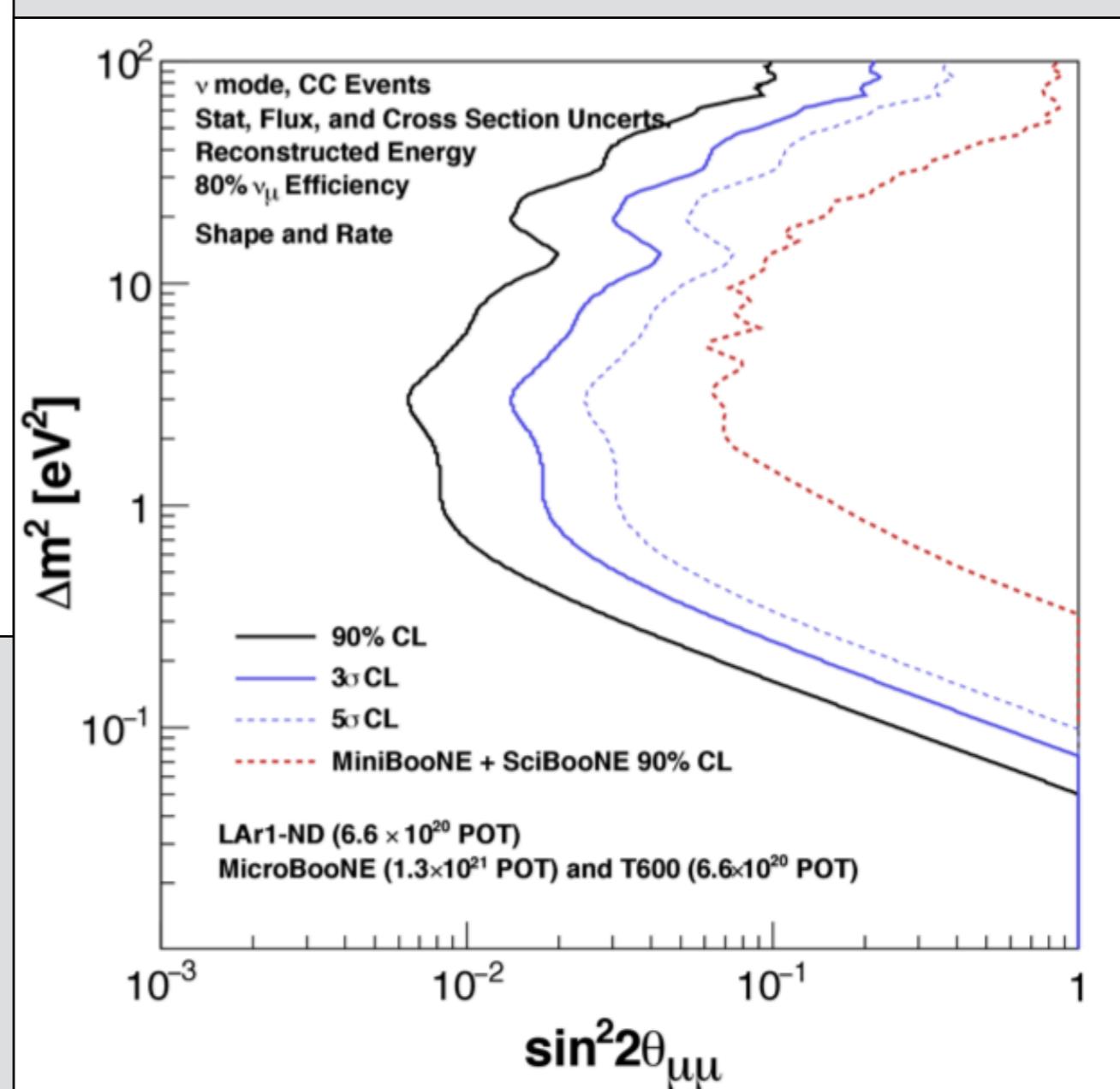
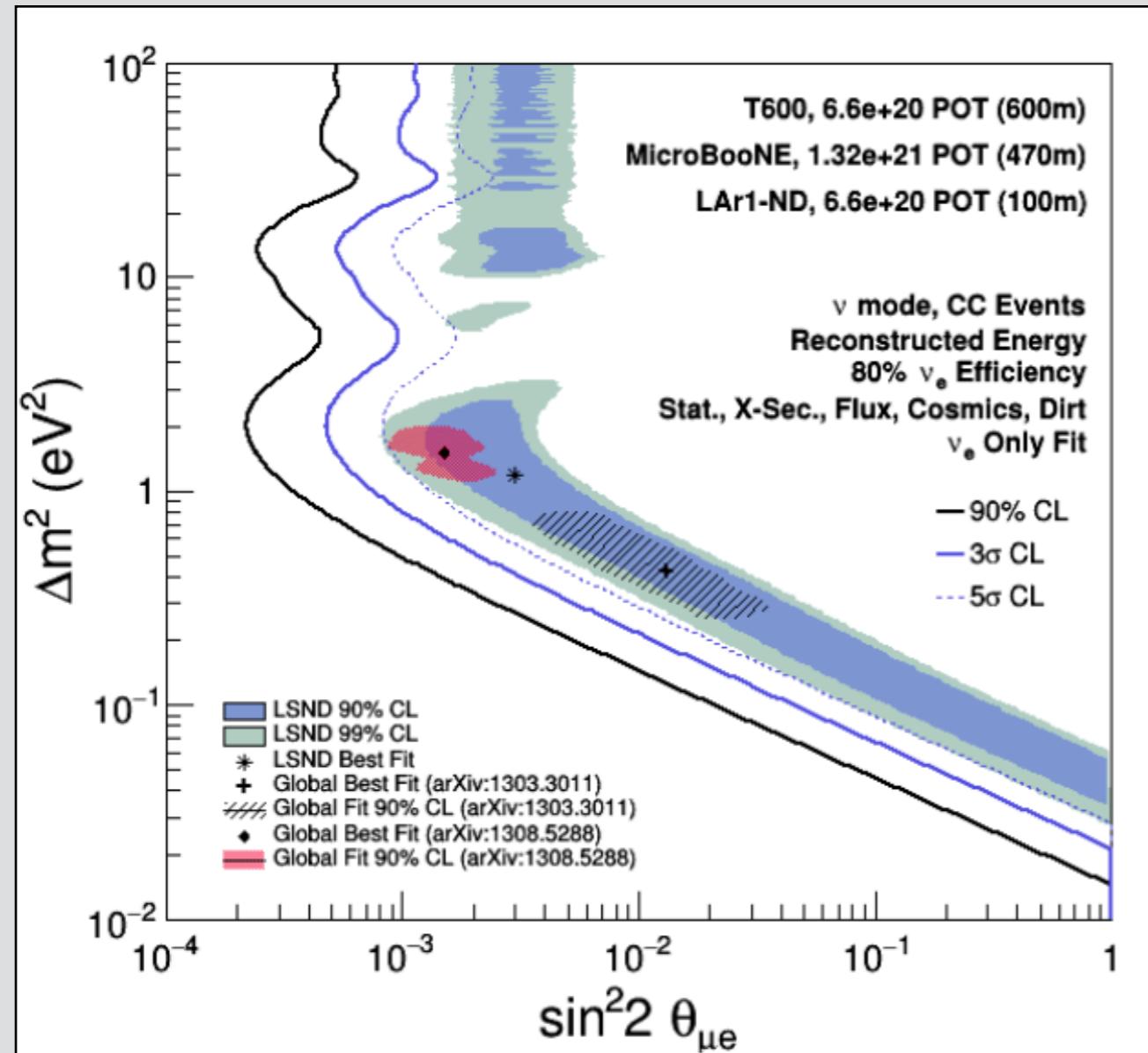


## 3+1 model at the SBNF



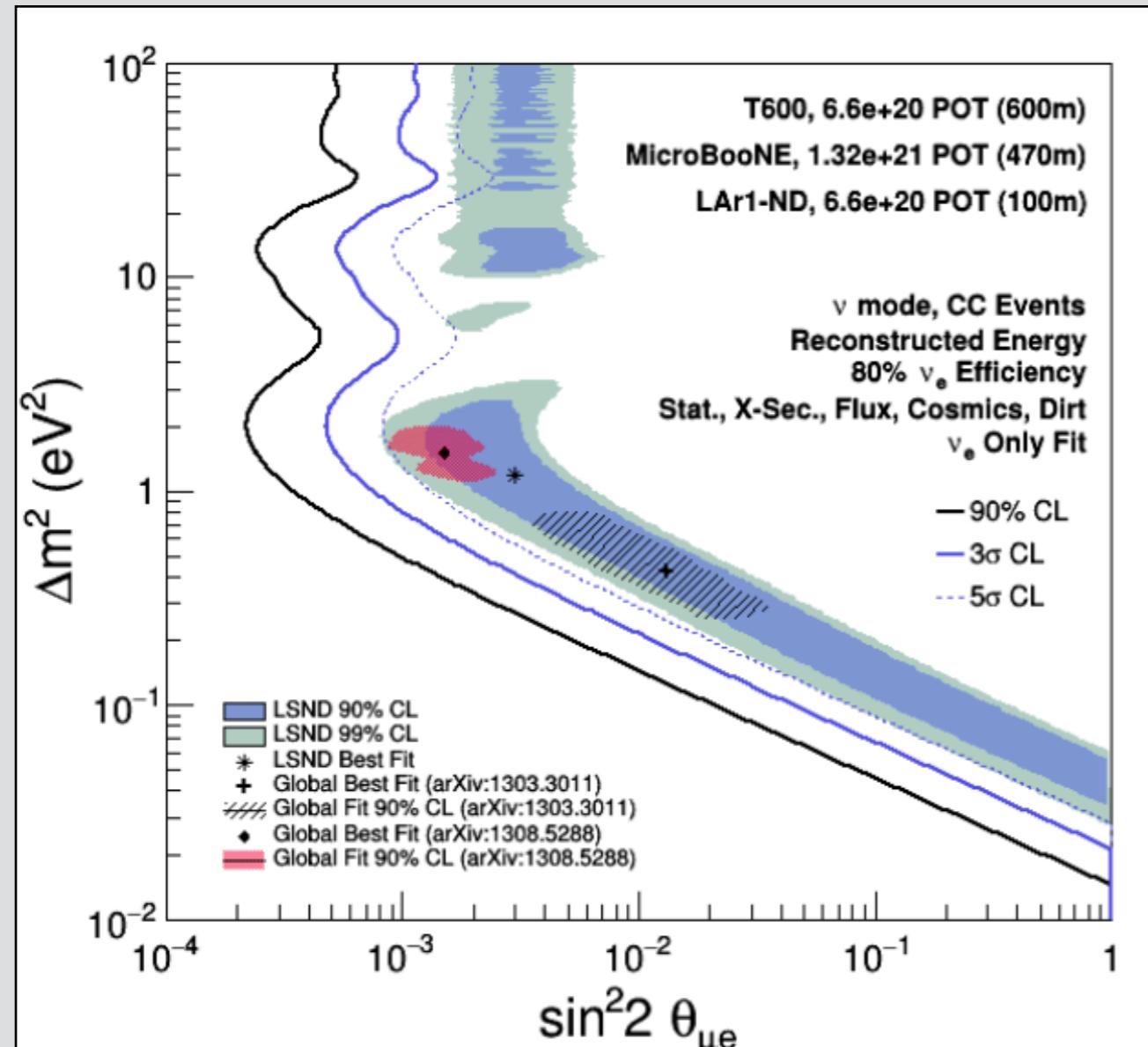
# Preliminary Results

## SBNF sensitivity curves for "3+1" model



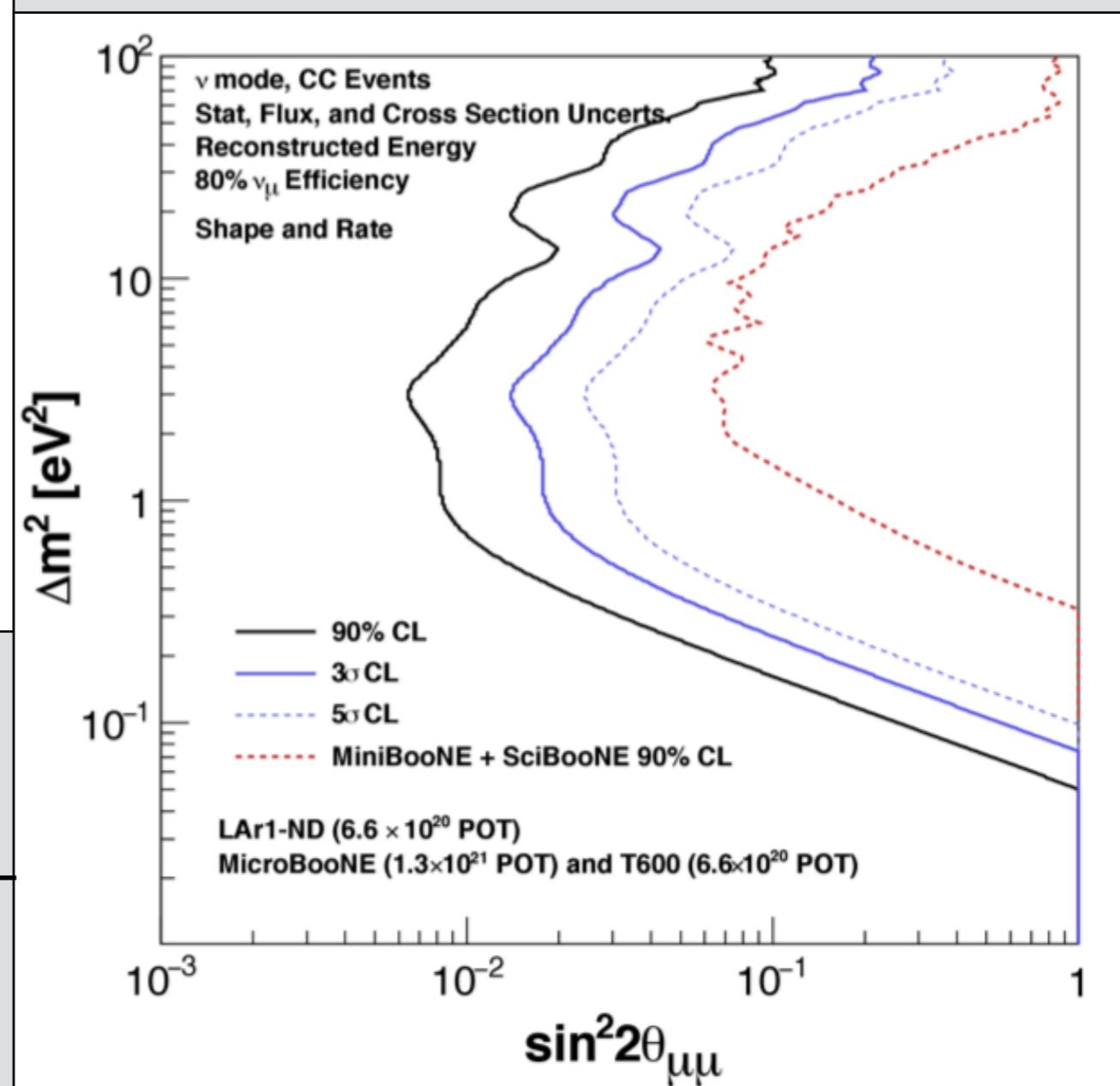
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arXiv:1503.01520

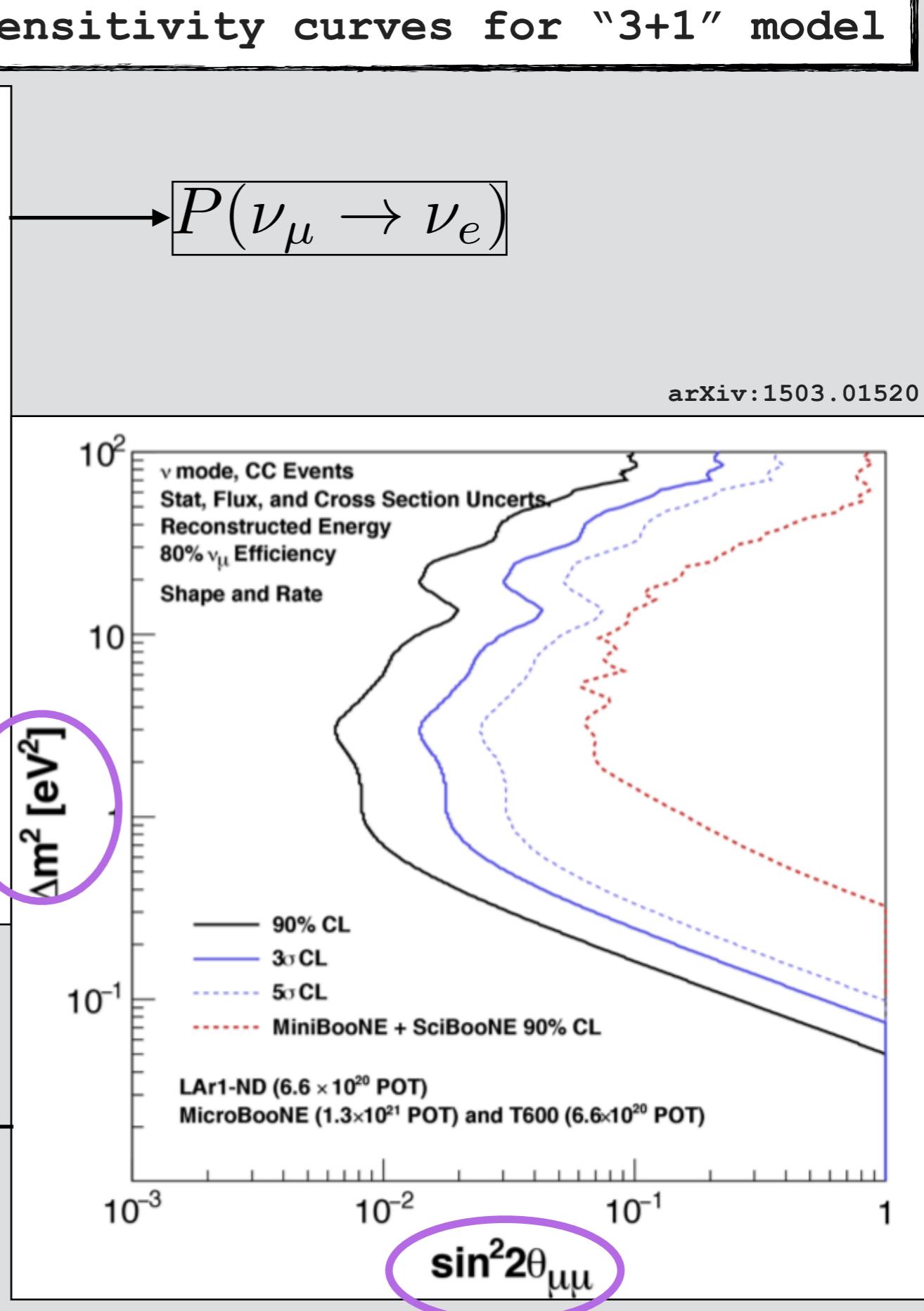
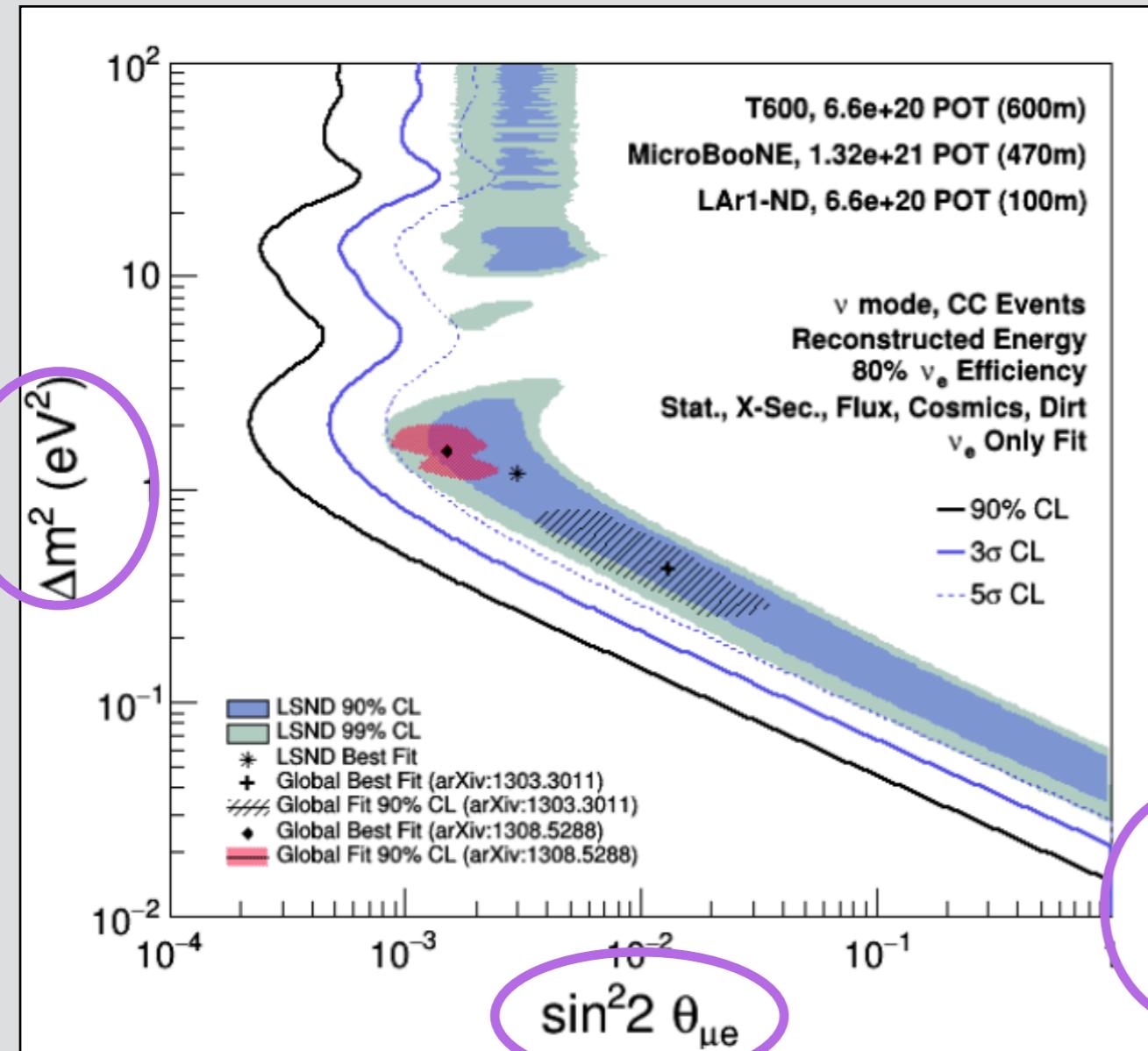
$$P(\nu_\mu \rightarrow \nu_\mu)$$



arXiv:1503.01520

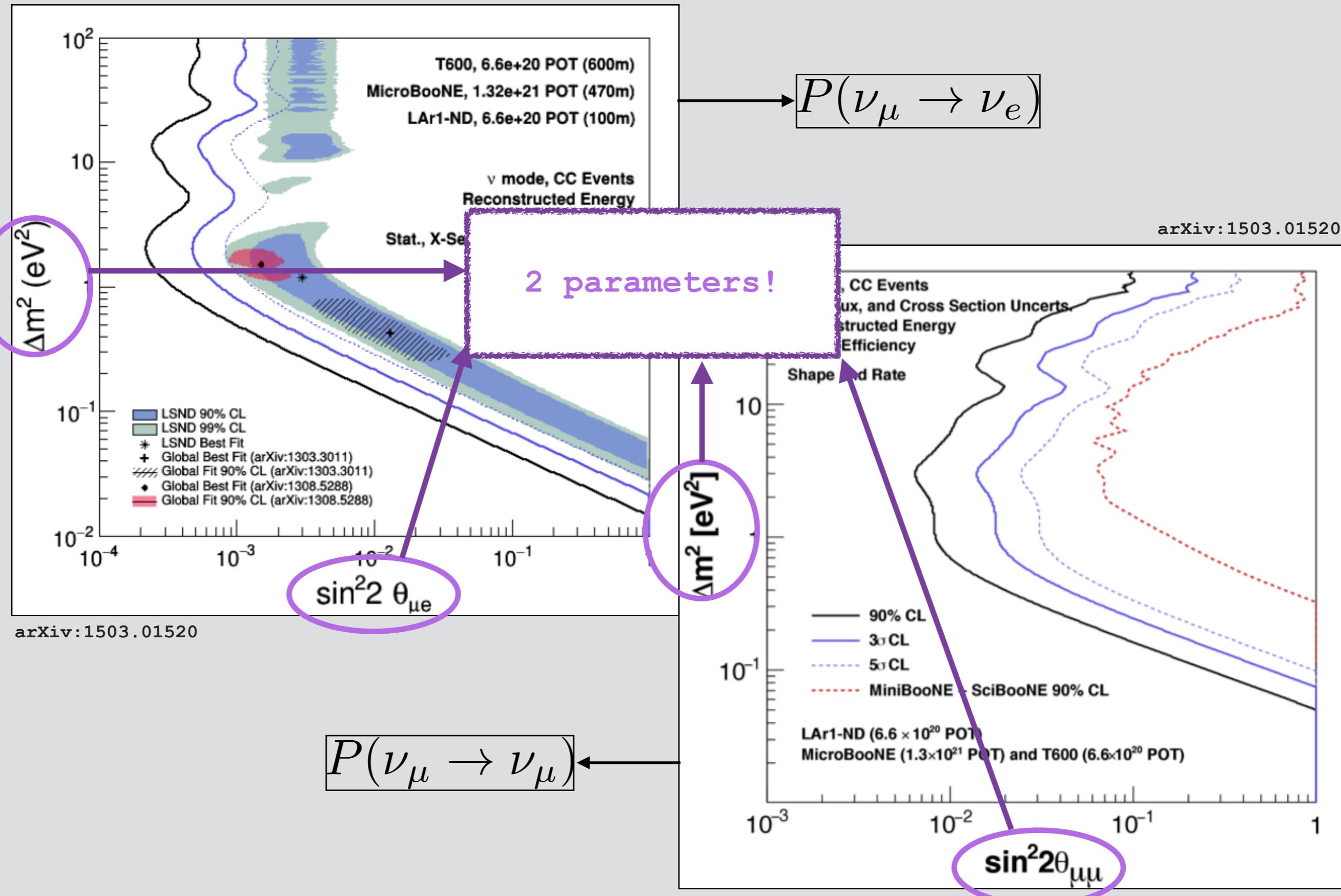
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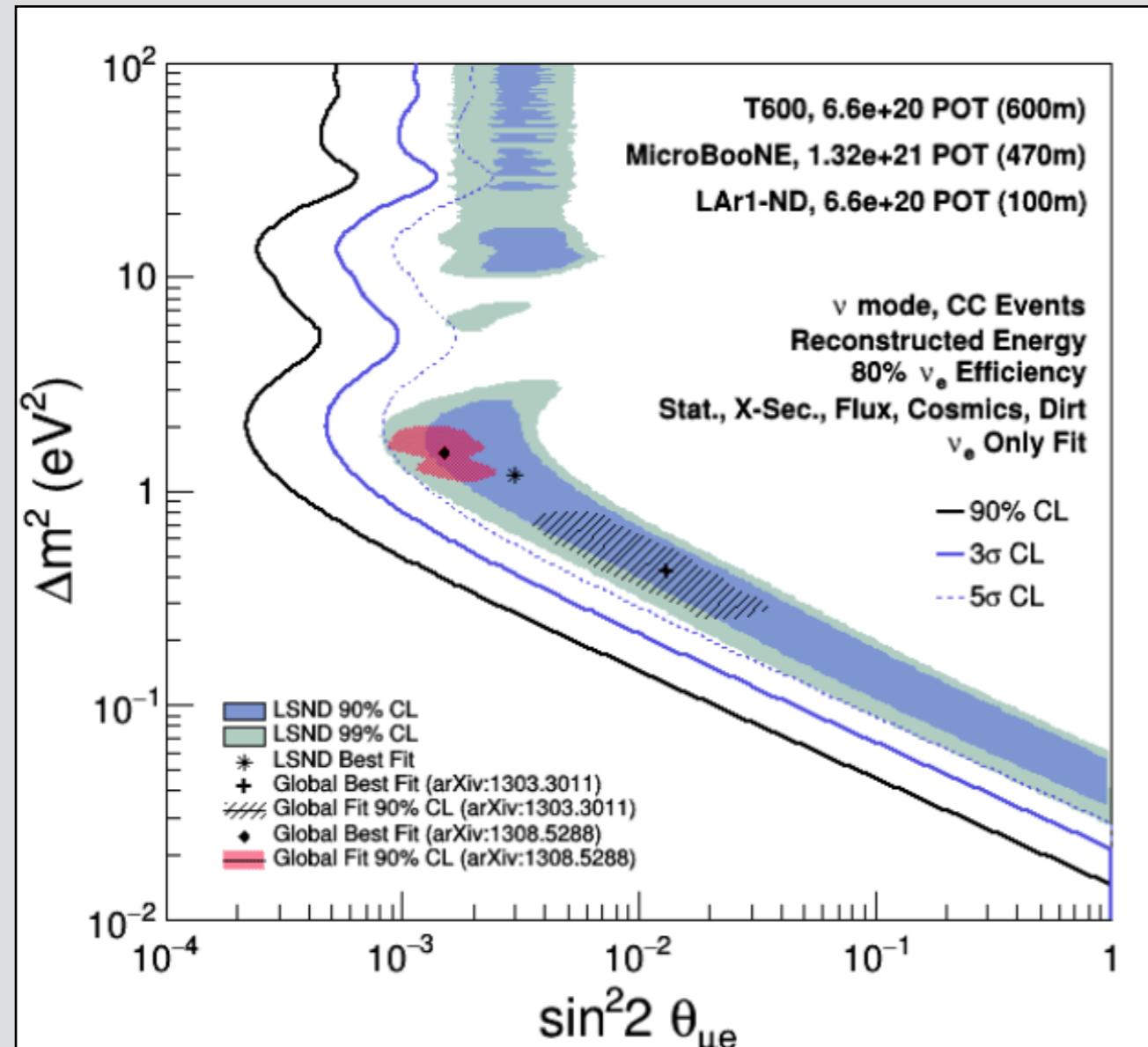
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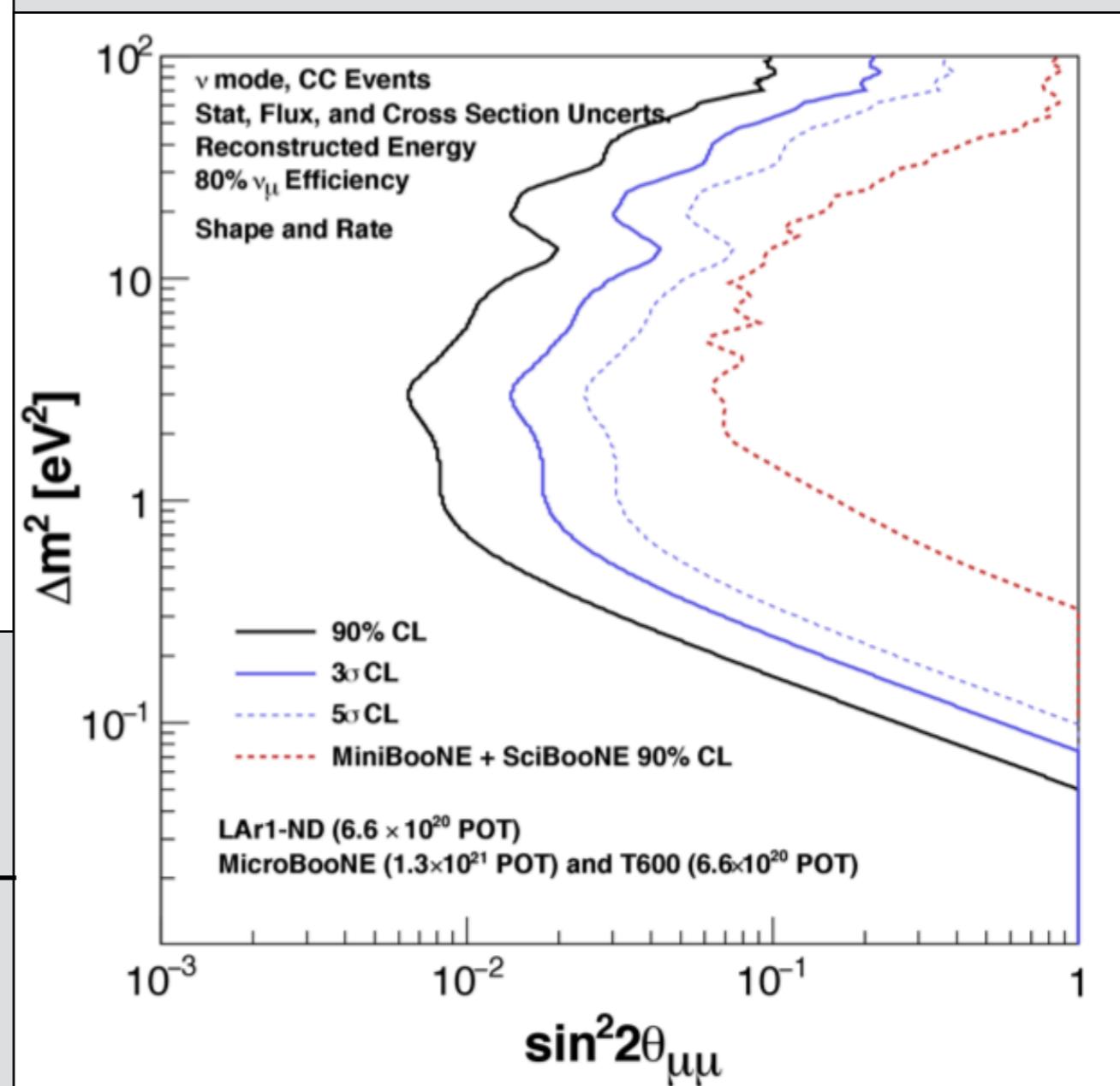
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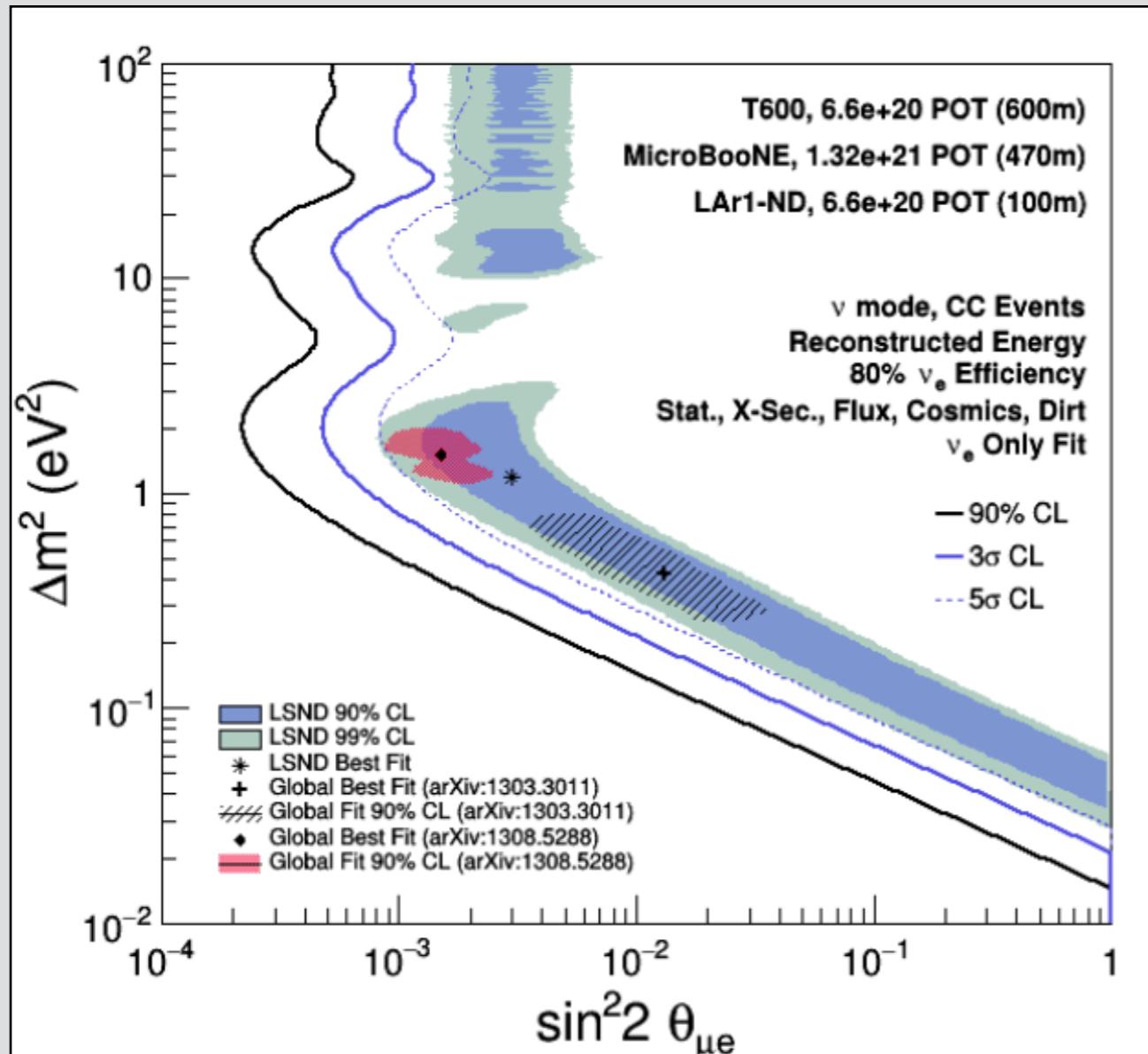
arXiv:1503.01520

$$P(\nu_\mu \rightarrow \nu_e)$$



# Preliminary Results

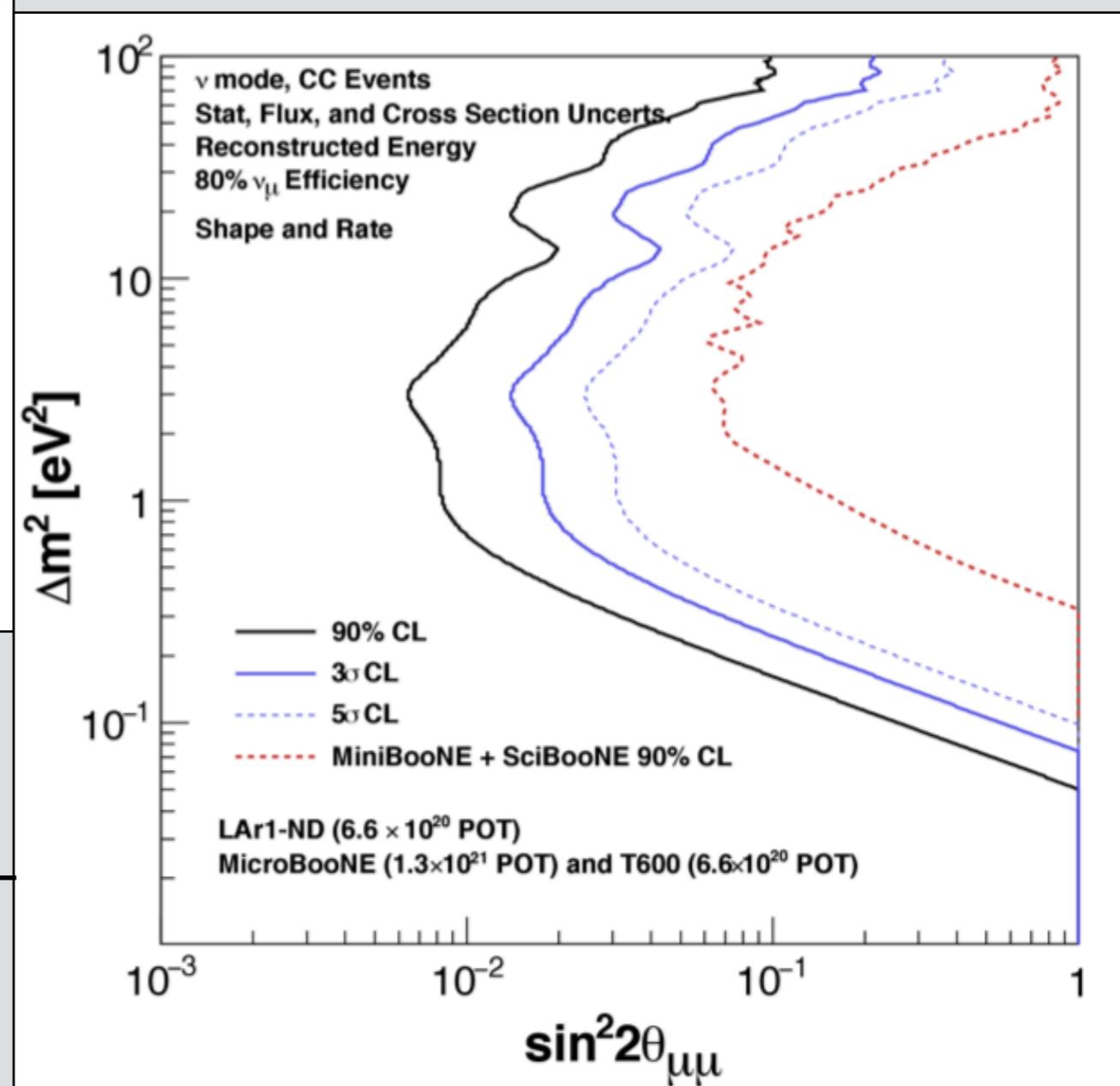
## SBNF sensitivity curves for "3+1" model



arXiv:1503.01520

$$(m_0^D, R_{ED})$$

$$P(\nu_\mu \rightarrow \nu_\mu)$$



# Preliminary Results

**There is no equivalence between LED and “3+1” model**

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**There is no equivalence between LED and “3+1” model**



**Short-baseline  
approximation with  
1 KK tower**

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Short-baseline  
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$$P(\nu_\mu \rightarrow \nu_\mu) = 1 - 4|U_{\mu 1}|^2 (S_1^{01})^2 \left( |U_{\mu 1}|^2 (S_1^{00})^2 + |U_{\mu 2}|^2 (S_2^{00})^2 + |U_{\mu 3}|^2 (S_3^{00})^2 \right) \sin^2 \left( 1.27 \frac{\lambda_1^{(1)2} - \lambda_1^{(0)2}}{R_{\text{ED}}^2} \frac{L}{E} \right)$$

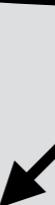
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$$P(\nu_\mu \rightarrow \nu_e) = ?$$

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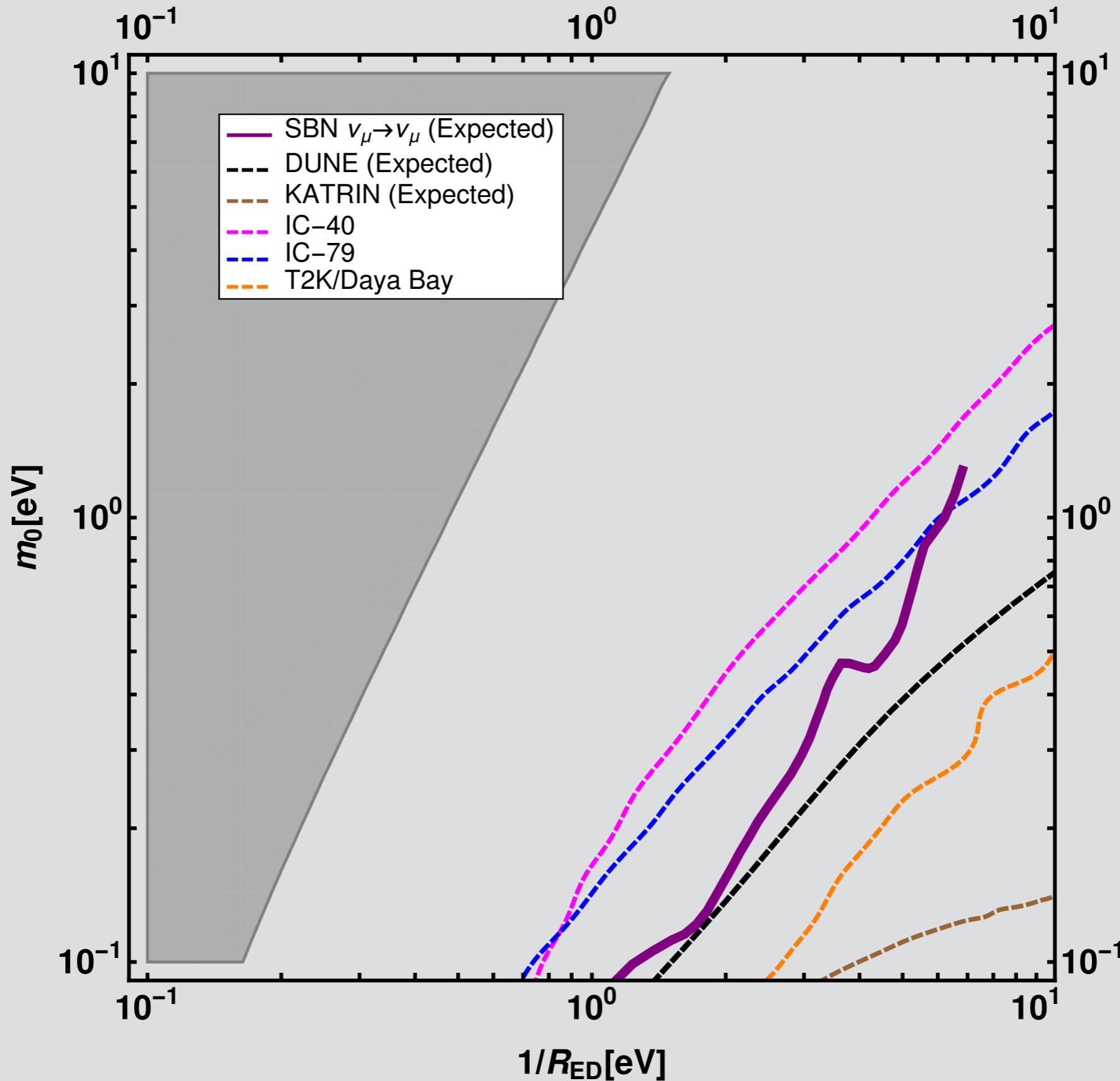
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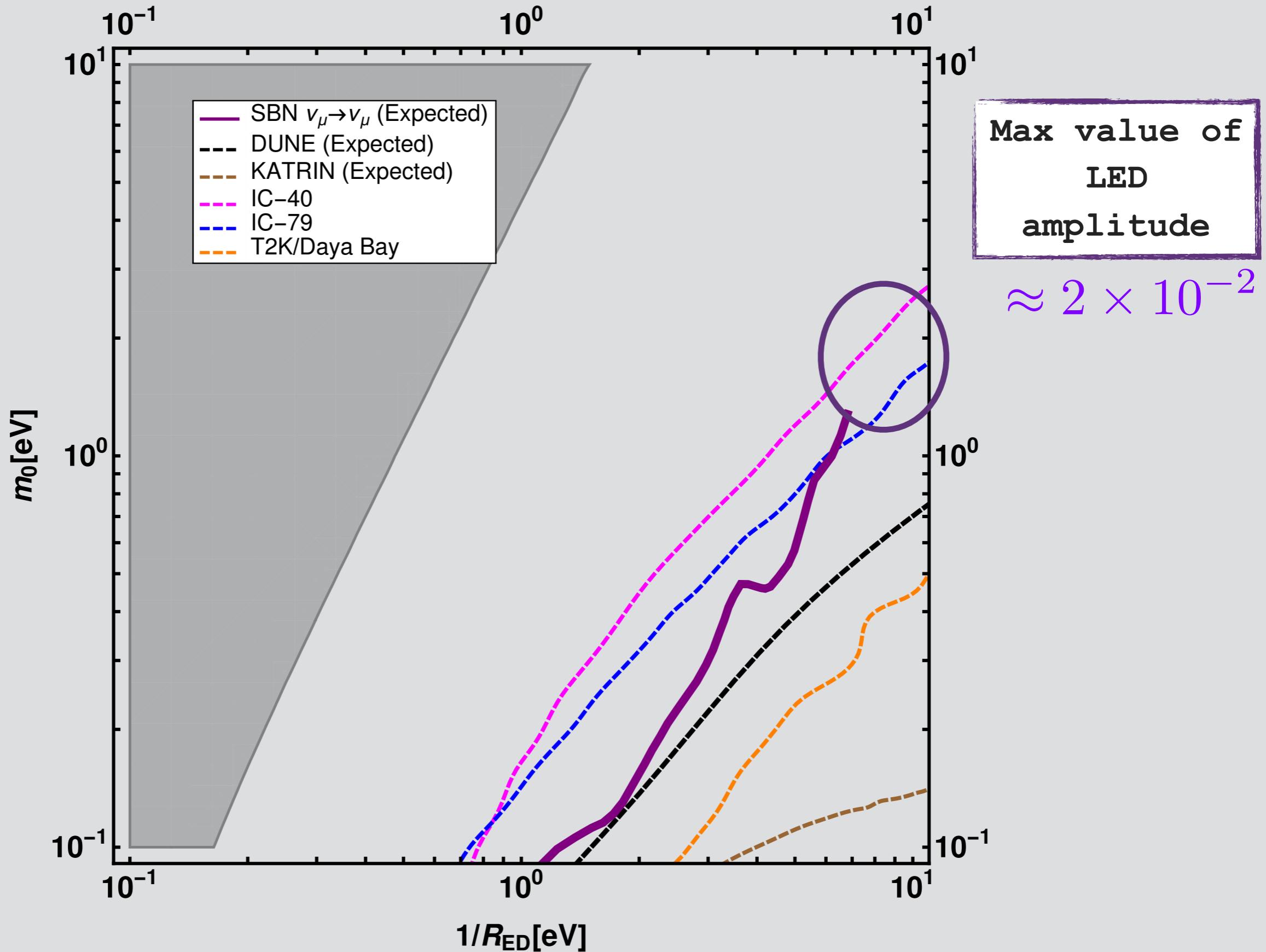
$$P(\nu_\mu \rightarrow \nu_e) = ? \quad \longrightarrow$$

Imaginary term in LED  
expression (CP-violation?)

# Preliminary Results



# Preliminary Results



# **What do we have for LED?**

We want to do the same analysis for LED

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ICARUS was calibrated  
in GLoBES (Same recipe  
for other two  
detectors) !

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LED has a numerical probability!

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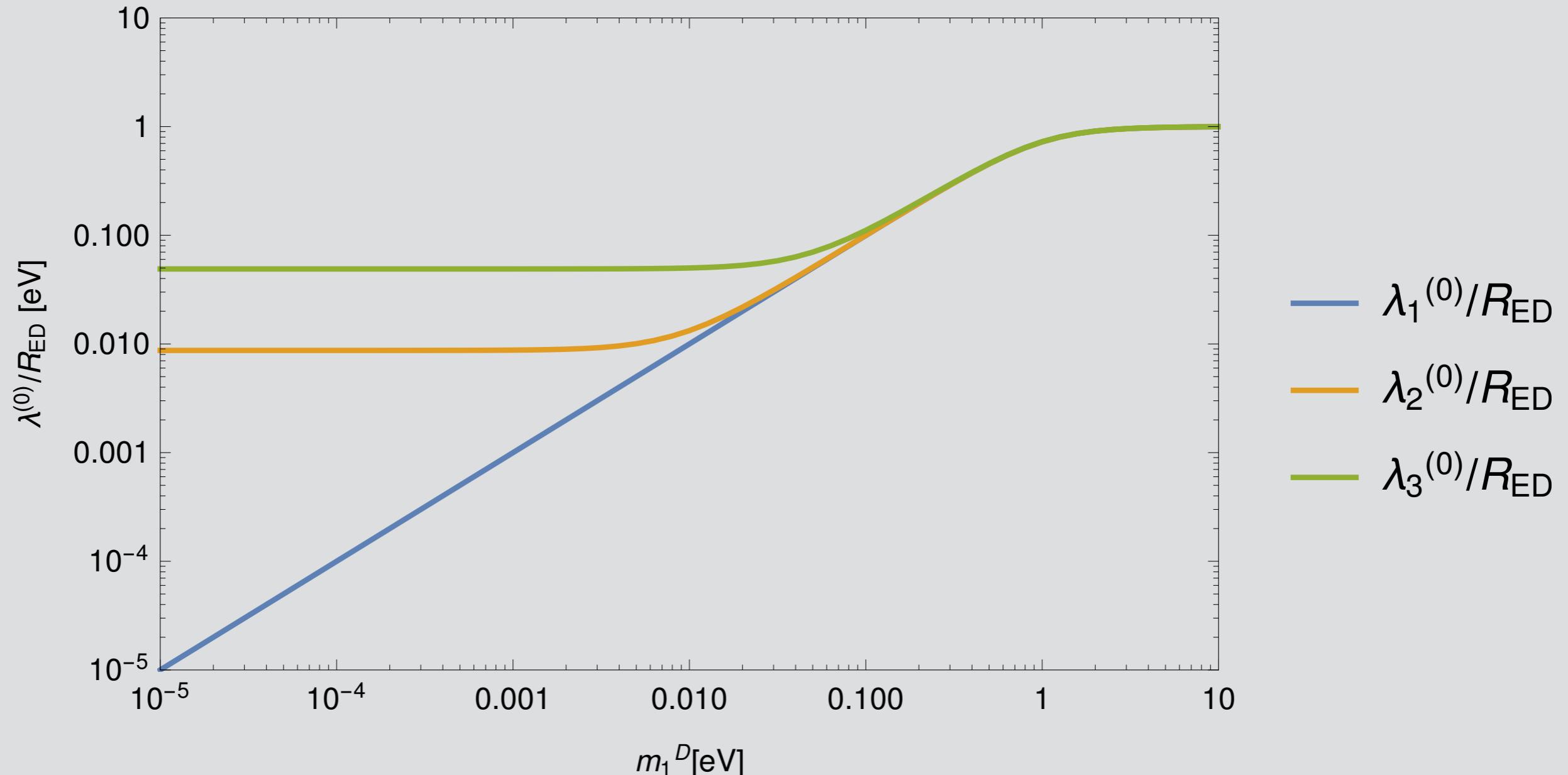


LED has a numerical probability!



How to implement it in  
GLoBES?

# Formalism



# Formalism

